UNIVERSITAT POLITÈCNICA DE CATALUNYA

DIAGNOSIS AND FAULT-TOLERANT CONTROL USING SET-BASED METHODS

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Advanced Control Systems (SAC)

Research Group

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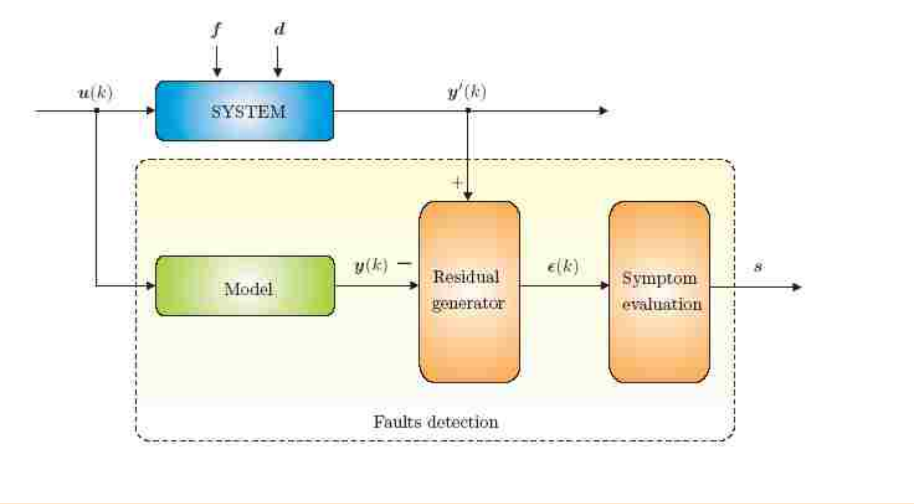
Model-based Fault Detection

- Model-based fault detection methods rely on the concept of **analytical redundancy**.



- However, modeling errors and disturbances in complex engineering systems are inevitable, and hence there is a need to develop robust fault detection algorithms.

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Robustness in Model-based Fault Detection

- The ***robustness*** of a fault detection system means that it must be only sensitive to faults, even in the presence of model-reality differences.

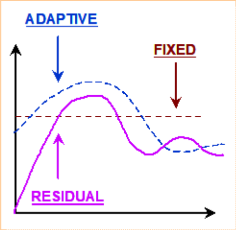
DETECTION PHASE

MEASURES → RESIDUAL GENERATION → DECISION MAKING → FAULT INDICATION

ACTIVE ROBUSTNESS

- ROBUST OBSERVERS
- ROBUST PARITY EQUATIONS

PASSIVE ROBUSTNESS



- ADAPTIVE THRESHOLD
- FUZZY LOGIC

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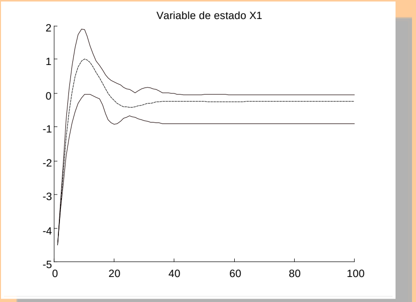
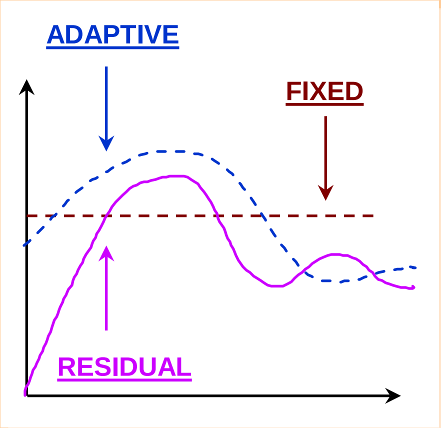
Passive Robust Decision-Making using Interval Models

RESIDUAL → DECISION MAKING → FAULT INDICATION

$$y_k \notin \hat{\Upsilon}_k = \left[\underline{\hat{y}}_k, \overline{\hat{y}}_k\right]$$



THRESHOLD



ADAPTIVE

STRUCTURED UNCERTAINTY → **INTERVAL MODELS AND METHODS**

UNSTRUCTURED UNCERTAINTY

(ENVELOPES)

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2. Interval Models for Fault Detection
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5. Identification for Robust Fault Detection
6. Fault-tolerance Evaluation
7. Real Applications
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Interval Model for FDI (1)

- Consider that the system to be monitored can be described by a general nonlinear model in discrete-time

$$\begin{aligned} x(k+1) &= f(x(k), u(k), \theta) \\ y(k) &= g(x(k), u(k), \theta) \end{aligned}$$

- The parameters $\theta 2 R^{m}$ are assumed to be unknown but belong to known intervals

$$\theta_i \in [\underline{\theta}_i, \overline{\theta}_i], \qquad i = 1 \ldots m$$

- An additional equation defining the allowed variance of parameters can be introduced for this purpose:

$$\theta(k+1) = \theta(k) + w(k)$$

where $|w(k)| \cdot \lambda$.

Interval Model for FDI (2)

- Measurement noise can be taken into account by assuming that the measurements are known to belong to intervals [*y(k)*], often created by adding an noise term σ to the actual measurement *y(k)*, that is,

$$[y(k)] = [y(k) - \sigma, y(k) + \sigma]$$

- In case uncertain parameters appear linearly with respect to inputs/outputs, the system model will be expressed in regressor form

$$y(k) = \varphi^T(k)\theta(k) + e(k)$$

- This corresponds to a MA parity equation.

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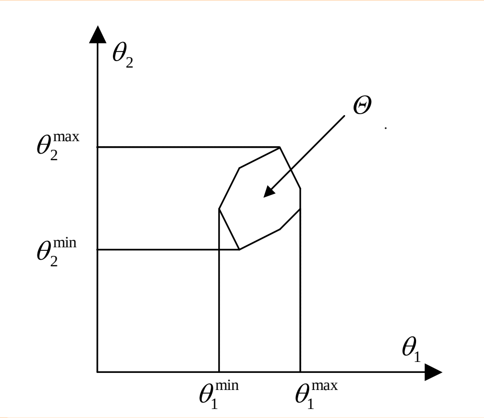
Fault Detection using Direct Image Test

- Considering the uncertainty in parameters $\boldsymbol{\theta} \in \boldsymbol{\Theta}$, the ***direct image test*** is

$$y(k) \in \left[\underline{\hat{y}}(k), \overline{\hat{y}}(k)\right]$$

Then, no fault is indicated. In other case, a fault is indicated.



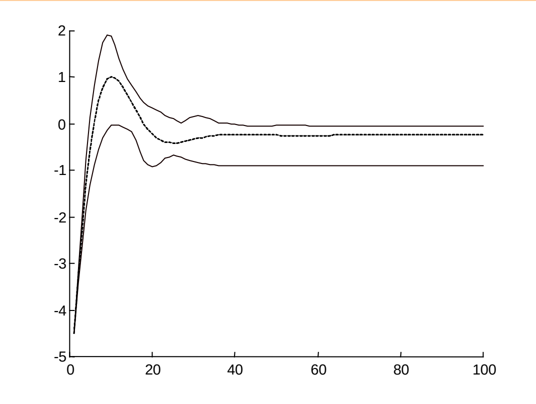
- The interval for the estimated output can be determined by

$$\boldsymbol{\varphi}^T(k)\underline{\boldsymbol{\theta}}(k) + \underline{\sigma} \le y(k) \le \boldsymbol{\varphi}^T(k)\overline{\boldsymbol{\theta}}(k) + \overline{\sigma}$$

where:

$$\underline{\boldsymbol{\theta}}(k) = \arg \min_{\boldsymbol{\theta} \in \mathcal{V}} \varphi^T \boldsymbol{\theta}$$

$$\overline{\boldsymbol{\theta}}(k) = \arg \max_{\boldsymbol{\theta} \in \mathcal{V}} \varphi^T \boldsymbol{\theta}$$

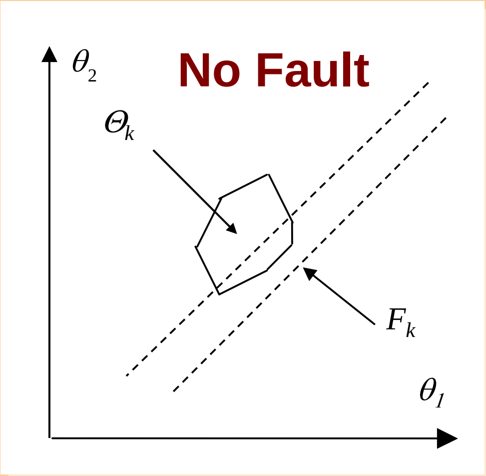


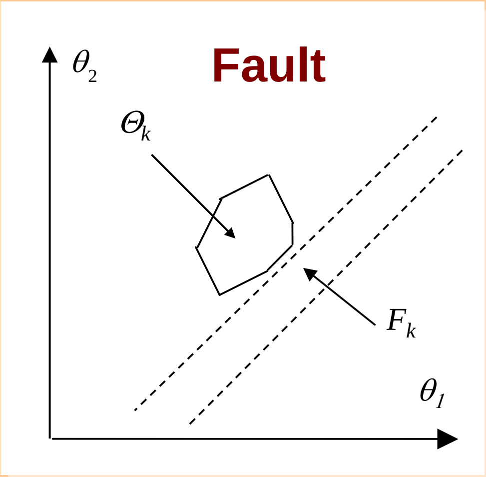
Fault Detection Algorithm using Inverse Test

$$\exists \boldsymbol{\theta} \in \boldsymbol{\Theta} \mid y(k) - \overline{\sigma} \leq \boldsymbol{\varphi}^T(k)\boldsymbol{\theta} \leq y(k) - \underline{\sigma}$$

$$F_k = \left\{ \theta \in \mathbb{R}^n : -\sigma \leq y(k) - \varphi(k)^T \theta \leq \sigma \right\}$$

$$F_k \cap \Theta_k \overset{?}{=} \varnothing$$





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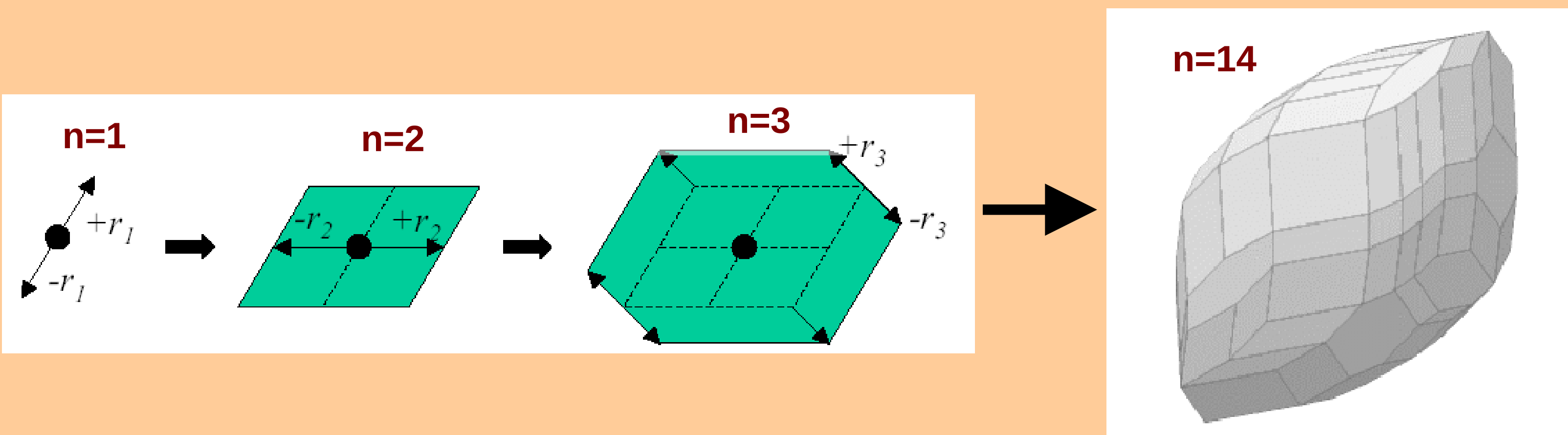
Zonotopes (1)

- A zonotope can be thought of as a ***Minkowski sum*** of a finite set of line segments:

$$\mathcal{X} = \mathbf{p} \oplus \mathbf{R}\mathbf{B}^m = \left\{ \mathbf{p} + \mathbf{R}\mathbf{z} : \mathbf{z} \in \mathbf{B}^m \right\}$$

- A zonotope can also be seen as the linear image of a *m-hypercube* in a *n-space*



Zonotopes (2)

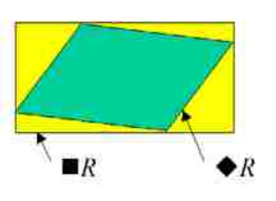
Zonotope Arithmetic

- Sum of two zonotopes: $\mathcal{X} = \mathbf{p} \oplus \mathbf{R}\mathbf{B}^m = (\mathbf{p}_1 + \mathbf{p}_2) \oplus \begin{bmatrix} \mathbf{R}_1 & \mathbf{R}_2 \end{bmatrix} \mathbf{B}^m$

- Image of a zonotope by a linear application ***L*:** $\mathcal{X} = (\mathbf{L}\mathbf{p}) \oplus (\mathbf{L}\mathbf{R})\mathbf{B}^m$

- Smallest interval box containing a zonotope ("interval hull"):

$$\square\mathcal{X} = \left\{ \mathbf{x} : \left| x_i - p_i \right| \leq \left\| \mathbf{R}_i \right\|_1 \right\}$$



- Inverse image of a zonotope by a linear application
- Intersection of two zonotopes

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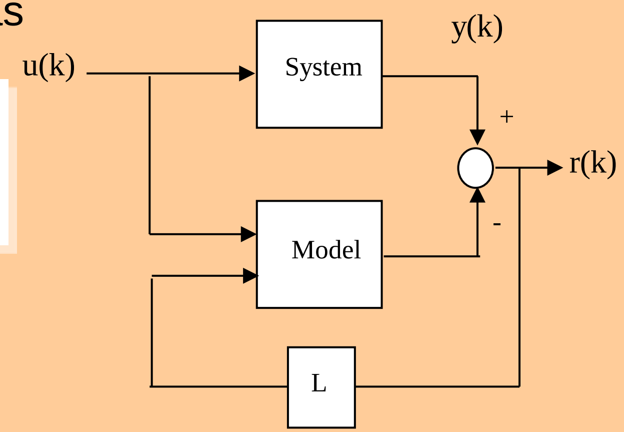
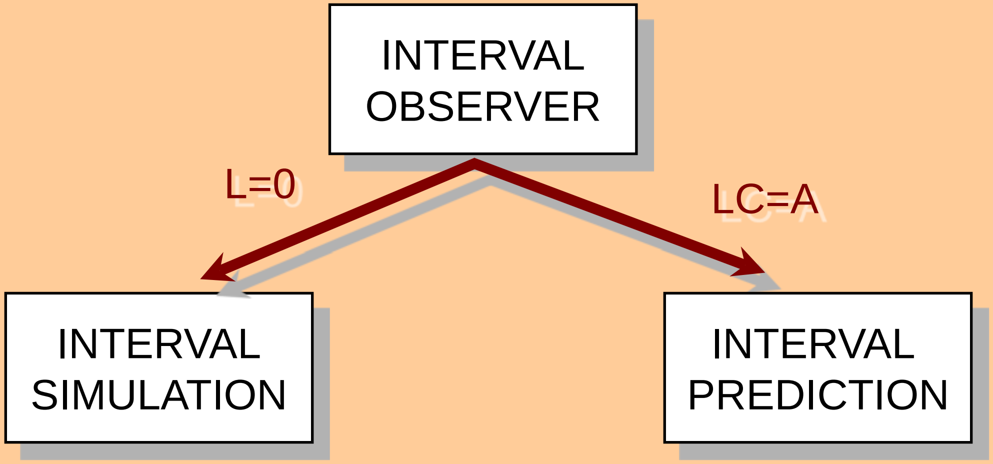
Interval Observer (1)

- Let the model for the state estimator of the monitored system described by a ***interval Luenberger observer*** formulated as

$$\begin{aligned}\hat{\mathbf{x}}_{k+1} &= \mathbf{A}(\boldsymbol{\theta})\hat{\mathbf{x}}_k + \mathbf{B}(\boldsymbol{\theta})\mathbf{u}_k + \mathbf{w}_k + \mathbf{L}(\mathbf{y}_k - \hat{\mathbf{y}}_k) \\ \hat{\mathbf{y}}_k &= \mathbf{C}\mathbf{x}_k + \mathbf{v}_k\end{aligned}$$



- This approach is in a half-way between ***simulation*** and ***prediction*** approaches.



Interval Observer (2)

- Let us denote the following sequences from the first time instant to time *k*:

$$\tilde{\boldsymbol{u}}_k = (\boldsymbol{u}_j)_0^{k-1} = (\boldsymbol{u}_0, \boldsymbol{u}_1, \cdots, \boldsymbol{u}_{k-1})$$
$$\tilde{\boldsymbol{y}}_k = (\boldsymbol{y}_j)_0^{k-1} = (\boldsymbol{y}_0, \boldsymbol{y}_1, \cdots, \boldsymbol{y}_k)$$
$$\tilde{\boldsymbol{w}}_k = (\boldsymbol{w}_j)_0^{k-1} = (\boldsymbol{w}_0, \boldsymbol{w}_1, \cdots, \boldsymbol{w}_{k-1})$$
$$\tilde{\boldsymbol{v}}_k = (\boldsymbol{v}_j)_0^{k-1} = (\boldsymbol{v}_0, \boldsymbol{v}_1, \cdots, \boldsymbol{v}_{k-1})$$
$$\tilde{\boldsymbol{\theta}}_k = (\boldsymbol{\theta}_j)_0^{k-1} = (\boldsymbol{\theta}_0, \boldsymbol{\theta}_1, \cdots, \boldsymbol{\theta}_{k-1})$$

- The set of estimated states at time *k* using the ***interval observer approach*** is expressed by

$$\hat{\mathcal{X}}_k = \left\{ \begin{array}{l} \hat{\boldsymbol{x}}_k \ \text{such that} \\ (\hat{\boldsymbol{x}}_{k+1} = \boldsymbol{A}(\boldsymbol{\theta}_k)\hat{\boldsymbol{x}}_k + \boldsymbol{B}\boldsymbol{u}_k + \boldsymbol{w}_k + \boldsymbol{L}(\boldsymbol{y}(k) - \hat{\boldsymbol{y}}(k)))_{j=1}^{k} \\ (\hat{\boldsymbol{y}}_k = \boldsymbol{C}\boldsymbol{x}_k + \boldsymbol{v}_k)_{j=0}^{k} \\ (\boldsymbol{w}_k \in \mathcal{W}, \boldsymbol{v}_k \in \mathcal{V}, \boldsymbol{\theta}_k \in \Theta)_{j=0}^{k}, \boldsymbol{x}_0 \in \mathcal{X} \end{array} \right\}$$

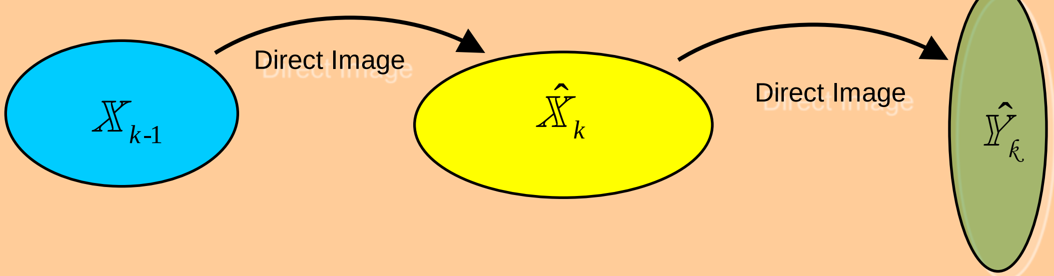
Implementation of Interval Observers

- The previous uncertain state set at time *k* can be computed approximately by admitting the rupture of the existing relations between variables of consecutive time instants:

Algorithm 1: Worst-case State Observer using Set Computations

1: $\hat{\mathcal{X}}_k \Leftarrow \mathcal{X}_0$
2: **for** $k = 1$ to N **do**
3: Compute $\hat{\mathcal{X}}_k$
4: Compute $\hat{\mathcal{Y}}_k$
5: **end for**

$$\boldsymbol{x}_{k+1} = \boldsymbol{A}(\theta)\boldsymbol{x}_k + \boldsymbol{B}(\theta)\boldsymbol{u}_k + \boldsymbol{w}_k + \boldsymbol{L}(\boldsymbol{y}_k - \hat{\boldsymbol{y}}_k) \qquad \boldsymbol{y}_k = \boldsymbol{C}\boldsymbol{x}_k + \boldsymbol{v}_k$$



Problems of Interval Observers

- When approximating the region of system states using sets several problems should be considered:
  - The ***wrapping effect***
  - The preservation of the ***parameter time-invariance***
  - The ***under/over estimation*** of the region
- These problems produce the ***propagation of the uncertainty***, deriving in the production of inconsistent, and even, unstable simulations/observations.

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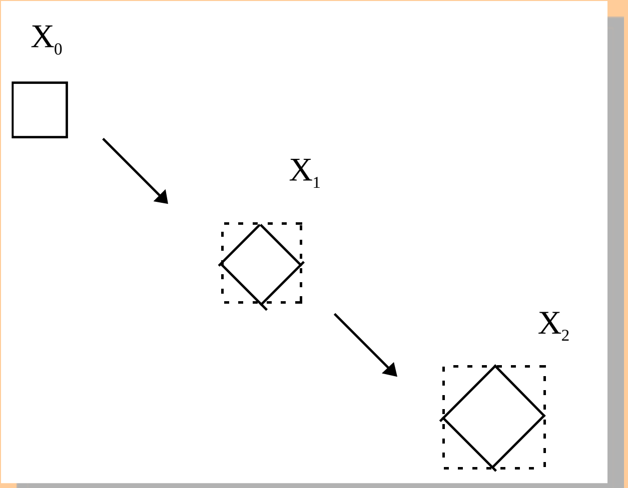
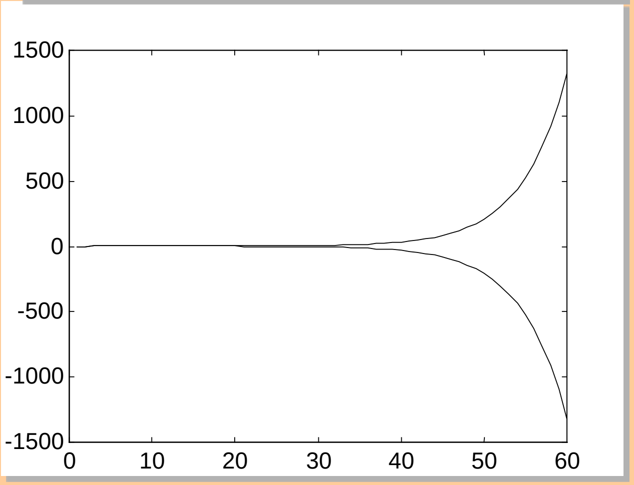
Wrapping Effect

- The ***problem of wrapping*** is related to the use of a crude approximation of the real region of state variables.
- At every stage of the simulation/observation, the ***true region*** of uncertain states ***is wrapped into a superset*** feasible to construct and to represent on a computer.
- Because of the overestimation of the a wrapped set is proportional to its radius, a ***spurious growth of the enclosures*** can result if the composition of wrapping and mapping is iterated.





Designing the Observer Gain to Avoid the Wrapping Effect

- Given a ***non-isotonic interval system***, an interval observer could be designed to fulfil the condition of isotonicity if all the elements of the observer matrix A_0 satisfy: $a_{ii}^{o} \geq 0$.

- In case of an isotonic observer is designed through appropriate selection of the observer gain, the wrapping effect is not present.

- Consequently, a simple iterative scheme based on a region propagation will work, providing the same results than a trajectory propagation algorithm.

- Moreover, a set-based (***time-varying***) interval observation and a trajectory based (***time-invariant***) interval observation will provide the same interval observation

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Fault Detection using Interval Observers (1)

- ***Fault detection test:***

  Given the sequences of measured inputs $\tilde{\boldsymbol{u}}_k$ and outputs $\tilde{\boldsymbol{y}}_k$ of the actual system, a ***fault*** is said to have occurred at time *k* if

$$y_k \notin \hat{\Upsilon}_k = \left[\underline{\hat{y}}_k, \overline{\hat{y}}_k \right]$$

or alternatively,

$$0 \notin \left[\underline{r}_k, \overline{r}_k \right] = y(k) - \left[\underline{\hat{y}}_k, \overline{\hat{y}}_k \right]$$

- In case noise in measurements is considered $y_k \in \Upsilon_k = \left[\underline{y}_k, \overline{y}_k \right]$, a fault is detected at time *k* if

$$\Upsilon_k \cap \hat{\Upsilon}_k = \varnothing$$

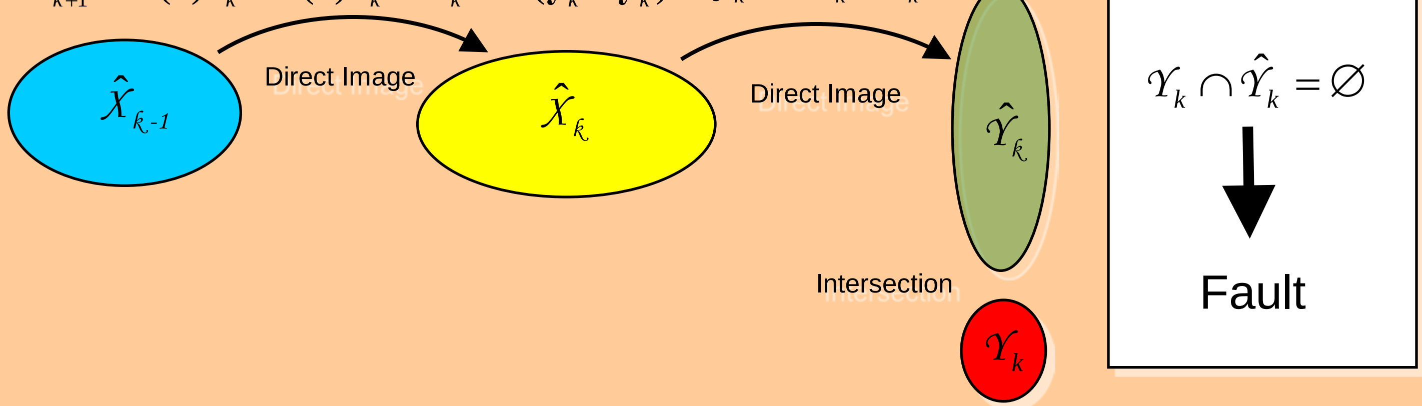
- ***Fault detection*** consists in detecting a fault using the previous test given a sequence of measured inputs $\tilde{\boldsymbol{u}}_k$ and ouptuts $\tilde{\boldsymbol{y}}_k$.

Fault Detection using Interval Observers (2)

Algorithm 2: Fault Detection using Worst-case Observer

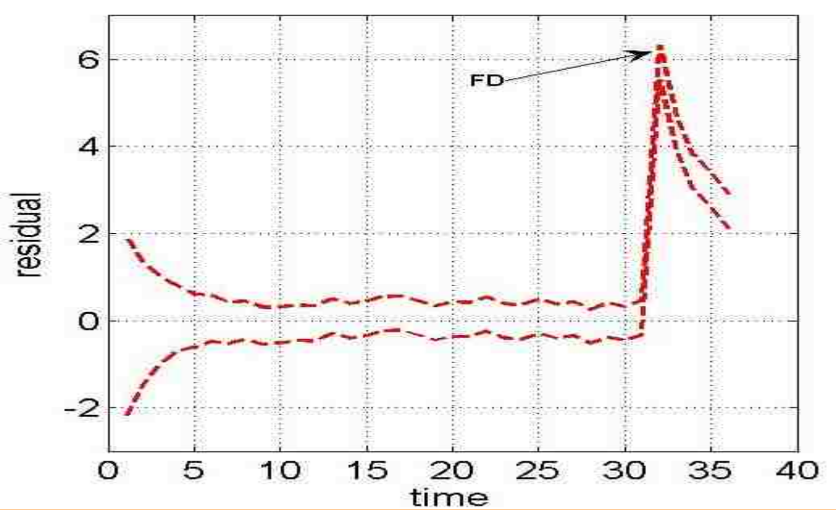
1: $\hat{\mathcal{X}}_k \Leftarrow \mathcal{X}_0$
2: **for** $k = 1$ to N **do**
3: Compute $\hat{\mathcal{X}}_k$
4: Compute $\hat{\mathcal{Y}}_k$
5: **if** $\hat{\mathcal{Y}}_k \cap \mathcal{Y}_k = \emptyset$ **then**
6: Exit (Fault detected)
7: **end if**
8: **end for**

$\mathbf{x}_{k+1} = \mathbf{A}(\boldsymbol{\theta})\mathbf{x}_k + \mathbf{B}(\boldsymbol{\theta})\mathbf{u}_k + \mathbf{w}_k + \mathbf{K}(\mathbf{y}_k - \hat{\mathbf{y}}_k)$ $\quad \mathbf{y}_k = \mathbf{C}\mathbf{x}_k + \mathbf{v}_k$

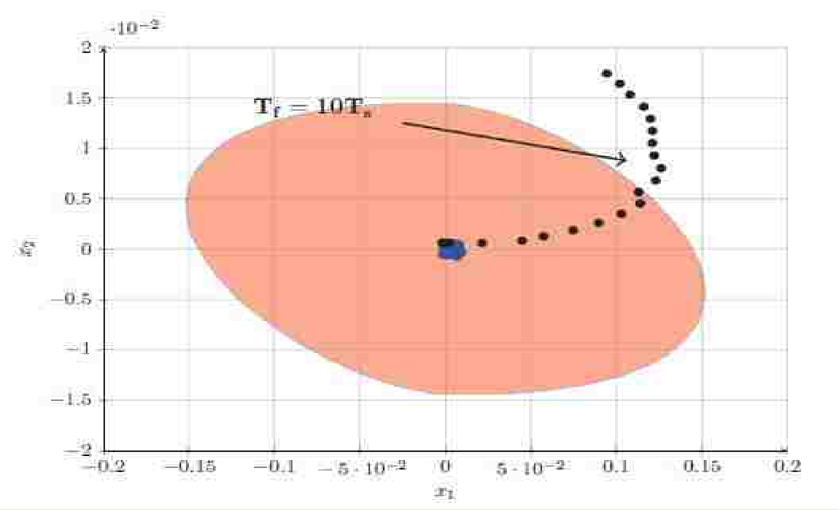


Invariant Sets and Interval Obsevers

Interval observer-based FD principle



Invariant set-based FD principle



M. Pourasghar, V. Puig, C. Ocampo-Martinez, Characterisation of interval-observer fault detection and isolation properties using the set-invariance approach, Journal of the Franklin Institute, Volume 357, Issue 3, 2020, Pages 1853-1886, https://doi.org/10.1016/j.jfranklin.2019.11.027.

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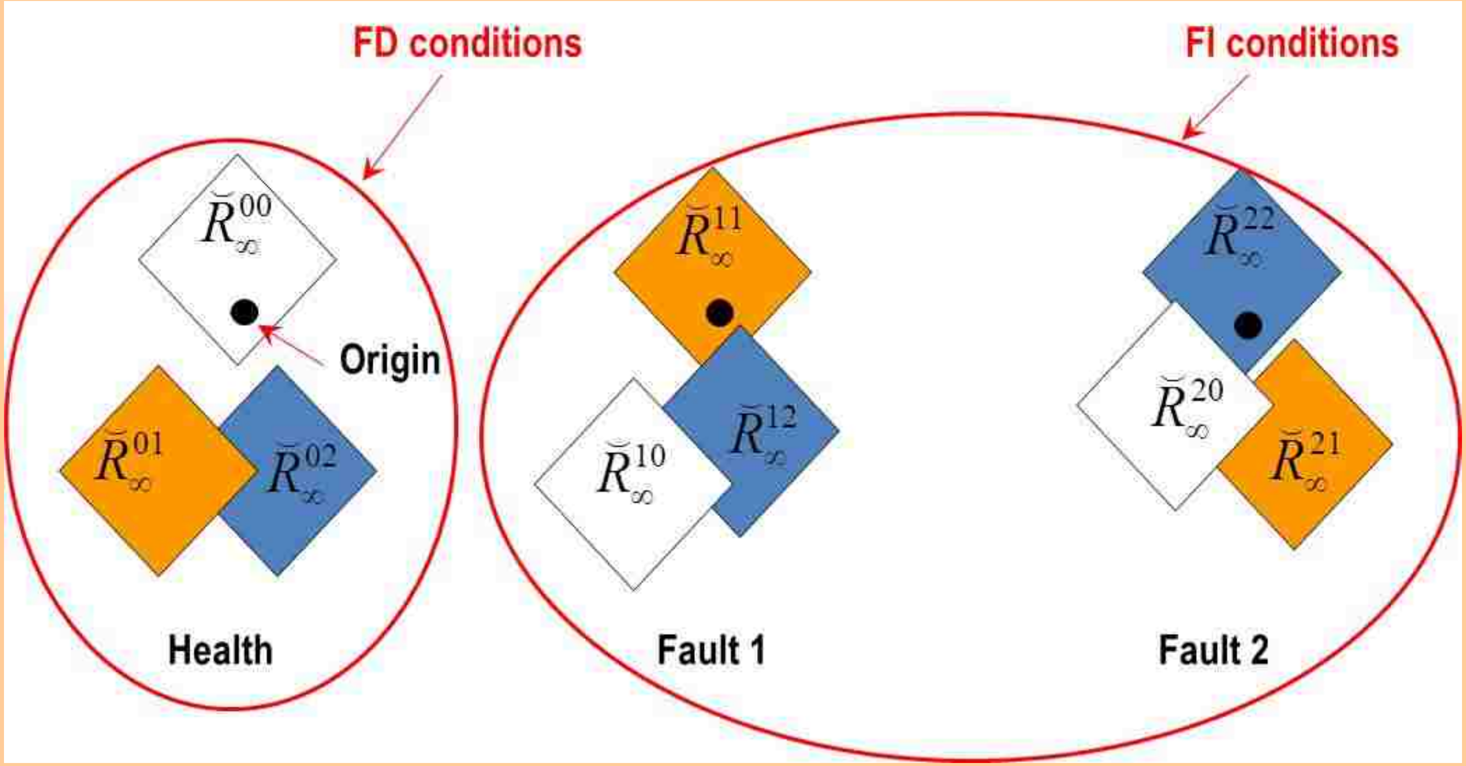
Advantages and Disadvantages

Invariant Sets	**Behaviors at steady state**
	Lower fault sensitivity (construct sets off-line)
	Lower complexity
Interval Observers	**System behaviors at transient and steady state**
	Higher fault sensitivity (estimate sets on-line)
	Higher complexity

Theoretical FDI Conditions

Theoretical FDI conditions :

$$0 \in \breve{R}_{\infty}^{ii} \text{ and } 0 \notin \breve{R}_{\infty}^{ij} \text{ for all } j \neq i$$



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Set-membership (or Consistency)-based Estimation Principle

- Let us denote the following sequences from the first time instant to time *k*:

$$\tilde{\boldsymbol{u}}_k = (\boldsymbol{u}_j)_0^{k-1} = (\boldsymbol{u}_0, \boldsymbol{u}_1, \cdots, \boldsymbol{u}_{k-1})$$
$$\tilde{\boldsymbol{y}}_k = (\boldsymbol{y}_j)_0^{k-1} = (\boldsymbol{y}_0, \boldsymbol{y}_1, \cdots, \boldsymbol{y}_k)$$
$$\tilde{\boldsymbol{w}}_k = (\boldsymbol{w}_j)_0^{k-1} = (\boldsymbol{w}_0, \boldsymbol{w}_1, \cdots, \boldsymbol{w}_{k-1})$$
$$\tilde{\boldsymbol{v}}_k = (\boldsymbol{v}_j)_0^{k-1} = (\boldsymbol{v}_0, \boldsymbol{v}_1, \cdots, \boldsymbol{v}_{k-1})$$
$$\tilde{\boldsymbol{\theta}}_k = (\boldsymbol{\theta}_j)_0^{k-1} = (\boldsymbol{\theta}_0, \boldsymbol{\theta}_1, \cdots, \boldsymbol{\theta}_{k-1})$$

- The set of estimated states at time *k* using the ***set-membership approach*** is expressed by

$$\mathcal{X}_k = \left\{ \begin{array}{l} \boldsymbol{x}_k \,\middle|\, \exists \tilde{\boldsymbol{w}}, \tilde{\boldsymbol{v}}, \tilde{\boldsymbol{\theta}}, \boldsymbol{x}_o \text{ such that} \\ (\boldsymbol{x}_{k+1} = \boldsymbol{A}(\boldsymbol{\theta}_k)\boldsymbol{x}_k + \boldsymbol{B}(\boldsymbol{\theta}_k)\boldsymbol{u}_k + \boldsymbol{w}_k)_{j=1}^{k} \\ (\boldsymbol{y}_k = \boldsymbol{C}\boldsymbol{x}_k + \boldsymbol{v}_k)_{j=0}^{k} \end{array} \right\}$$

Puig, V. (2010). Fault diagnosis and fault tolerant control using set-membership approaches: Application to real case studies, *International Journal of Applied Mathematics and Computer Science*, *20*(4), 619-635. doi: https://doi.org/10.2478/v10006-010-0046-y

Implementation of Set-membership Estimators (1)

- The previous uncertain state set at time *k* can be computed approximately by admitting the rupture of the existing relations between variables of consecutive time instants.

- Two sets are introduced:

  - The ***set of predicted states*** at time *k* is given by

$$\mathbb{X}_k^p = \mathbf{x}_k : \overline{\mathbf{A}}(\theta_{k-1})\mathbf{x}_{k-1} + \overline{\mathbf{B}}(\theta_{k-1})\mathbf{u}_{k-1} + \overline{\mathbf{E}}\mathbf{y}_k + \overline{\mathbf{w}}_{k-1} | \\ \mathbf{x}_{k-1} \in \mathbb{X}_{k-1}, \theta_k \in \Theta, \overline{\mathbf{w}}_{k-1} \in \overline{\mathbb{W}}_{k-1} \Big\}$$

  - The ***set of consistent states*** at time *k* with measurement is defined as

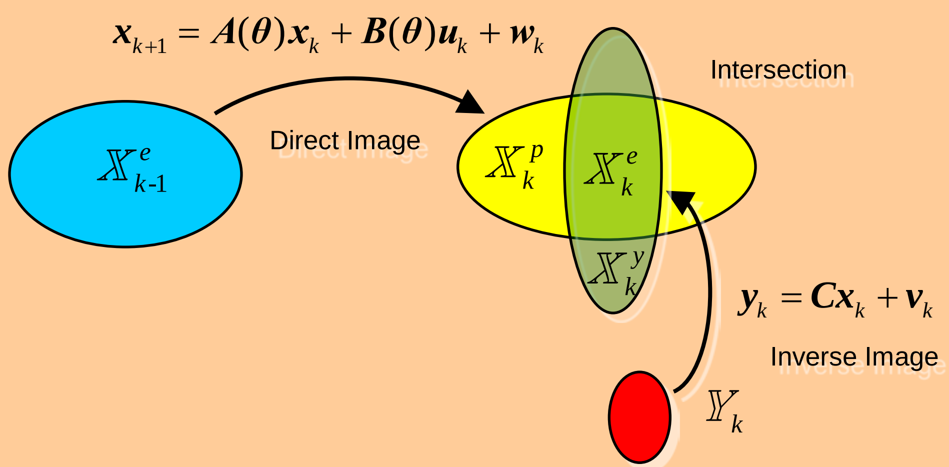
$$\mathbb{X}_k^{y_k} = \Big\{ \mathbf{x}_k \, : \, \mathbf{y}_k = \overline{\mathbf{C}}\mathbf{x}_k + \overline{\mathbf{v}}_k, \; \theta_k \in \Theta, \overline{\mathbf{v}}_k \in \overline{\mathbb{V}}_k \Big\}$$

Implementation of Set-membership Estimators (2)

- This allows to write the following algorithm:

Algorithm 1: Set-membership State Estimation using Set Computations

$\mathcal{X}_k^e \Leftarrow \mathcal{X}_0$
for $k = 1$ to N **do**
  Compute $\mathcal{X}_k^p$
  Compute $\mathcal{X}_k^c$
  Compute $\mathcal{X}_k^e = \mathcal{X}_k^p \cap \mathcal{X}_k^c$
end for



Fault Detection using Set-membership Estimation (1)

- ***Fault detection test:***

  Given the sequences of measured inputs $\tilde{\boldsymbol{u}}_k$ and outputs $\tilde{\boldsymbol{y}}_k$ of the actual system, a **fault** is said to have occurred at time *k* if there does not exist a set of sequences ($\tilde{\boldsymbol{w}}_k, \tilde{\boldsymbol{v}}_k, \tilde{\boldsymbol{\theta}}_k$) which satisfy the nominal system description with initial condition, noise, disturbances and parameters belonging to ($\mathcal{X}_o, \mathcal{V}, \mathcal{W}, \Theta$), respectively.

- ***Fault detection*** consists in detecting a fault given a sequence of measured inputs $\tilde{\boldsymbol{u}}_k$ and outputs $\tilde{\boldsymbol{y}}_k$.

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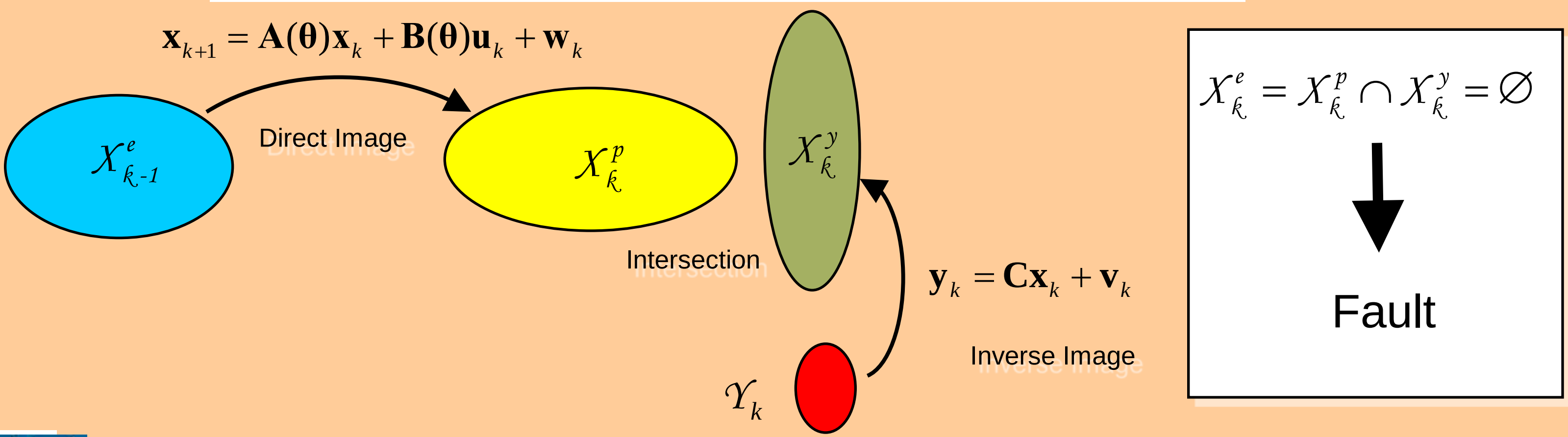
Fault Detection using Set-membership Estimation (2)

Algorithm 2: Fault Detection using Set-membership Estimation

$\mathcal{X}_k^e \Leftarrow \mathcal{X}_0$
for $k = 1$ to N **do**
  Compute $\mathcal{X}_k^p$
  Compute $\mathcal{X}_k^c$
  Compute $\mathcal{X}_k^e = \mathcal{X}_k^p \cap \mathcal{X}_k^c$
  if $\mathcal{X}_k^e = \emptyset$ **then**
    Exit (Fault detected)
  end if
end for



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Identification for Robust Fault Detection

- One of the key points in model based fault detection is how detection models are estimated.
- In case of set-membership methods, the set for uncertain parameters should be estimated.
- The set for uncertain parameters depend on the way how the uncertain model will be used for fault detection.
- At least two possible types of models can be derived:
  - interval model
  - set-membership or consistency based model

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Blesa, J., Puig, V. and Saludes, J. (2011), Identification for passive robust fault detection using zonotope-based set-membership approaches. Int. J. Adapt. Control Signal Process., 25: 788-812. https://doi.org/10.1002/acs.1242

Identification for the Direct Test (1)

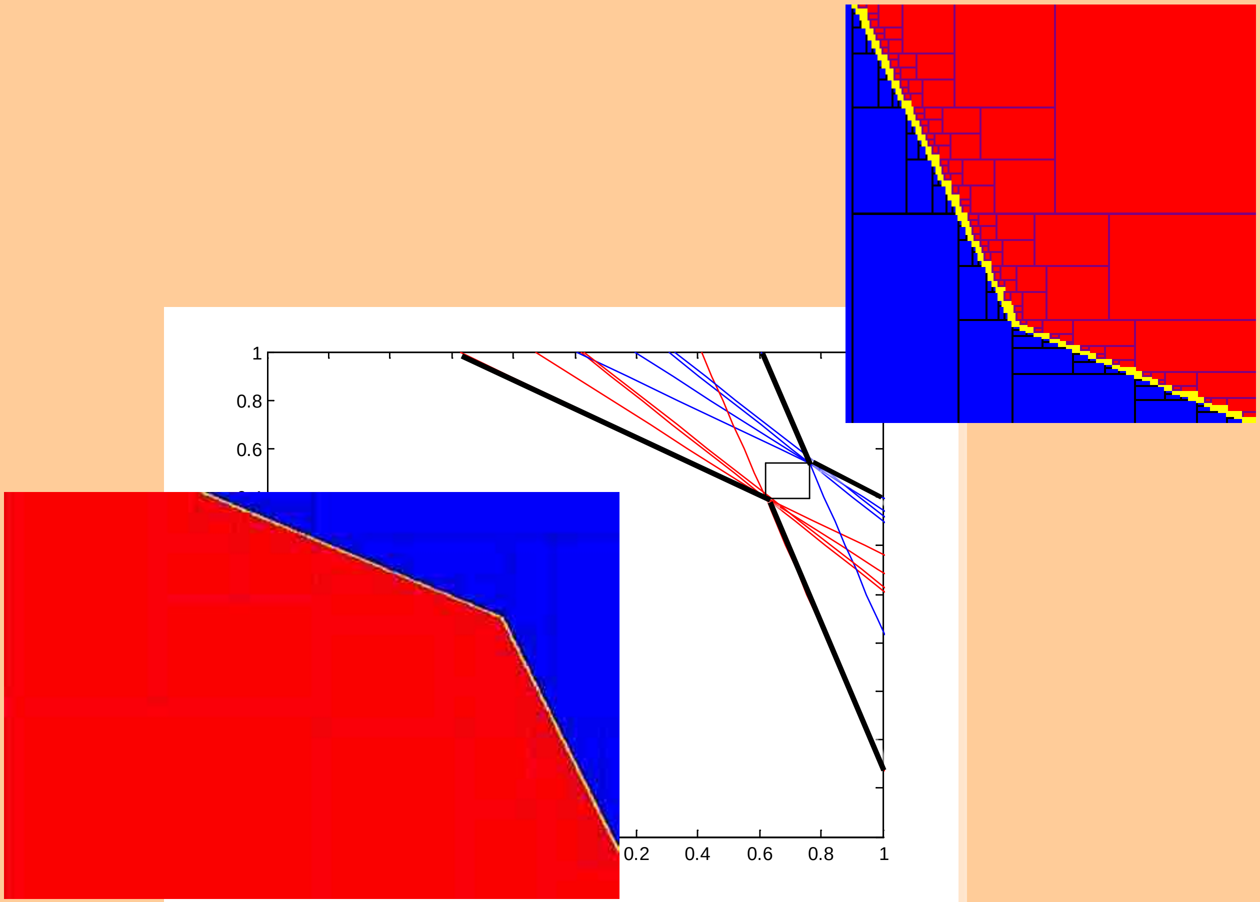
Given a set of measurements $y(k)$ taken in a given interval $k \in [0, N]$, considering that noise is bounded such that $y_m(k) \in Y_m(k)$, then a set of model parameters that produces an envelope that cover all measurements ("**worst-case approach**"):

$$\Theta = \left\{ \theta \in \Theta \mid \forall y(k) \in Y(k), \forall k \in [0, N] \quad (\underline{y}(k,\theta) \le y(k)) \wedge (y(k) \le \overline{y}(k,\theta)) \right\}$$

where at each time tinstant k, model temporal envelope is computed according to:

$$\begin{aligned} &\underline{y}(t_k) = min\, y(t_k, \theta) \\ &sujeto\ a: \\ &\theta \in \Theta \end{aligned} \qquad\qquad \begin{aligned} &\overline{y}(t_k) = max(t_k, \theta) \\ &sujeto\ a: \\ &\theta \in \Theta \end{aligned}$$

Identification for the Direct Test (2)



Identification for the Inverse Test (1)

Given a set of measurements $y(k)$ taken in a given interval $k \in [0, N]$, considering that noise is bounded such that $y_m(t) \in Y_m(t)$, then a set of model parameters that are consistent with model and measurements would be estimated such that ("**consistency approach**"):

$$\Theta = \left\{ \theta \in \Theta \mid \exists y(k) \in Y(k), \forall k \in [0, N] \quad \underline{y}(k) \leq y(k, \theta) \leq \overline{y}(k) \right\}$$

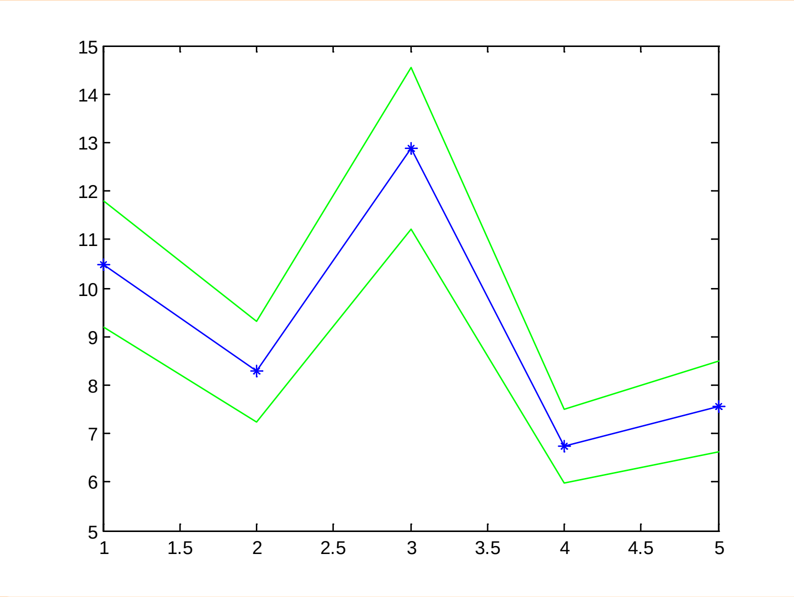
This set can be computed at each sample time instant k:

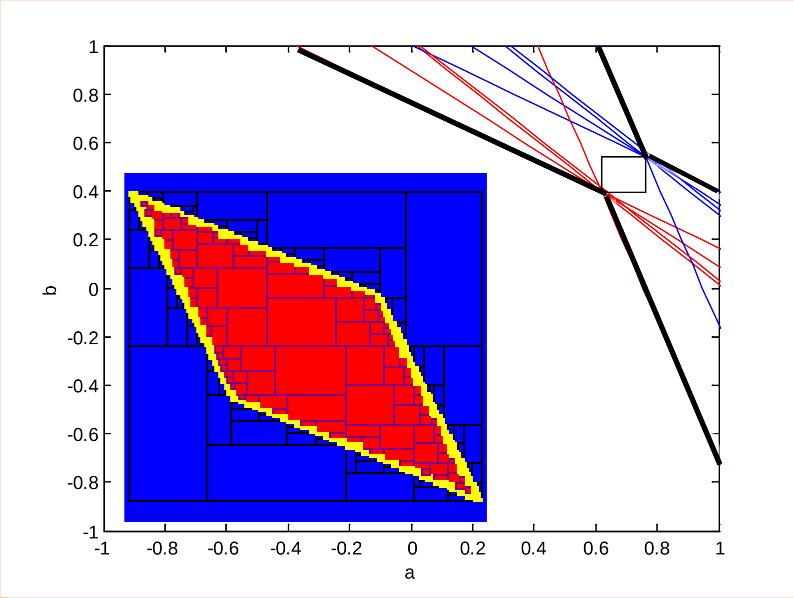
$$\Theta = \left\{ \theta \in \Theta \mid \exists y(k) \in Y(k) \quad \underline{y}(k) \leq y(k, \theta) \leq \overline{y}(k) \right\}$$

such that:

$$\Theta = \bigcap_{k=1}^{N} \Theta_k$$

Identification for the Inverse Test (2)





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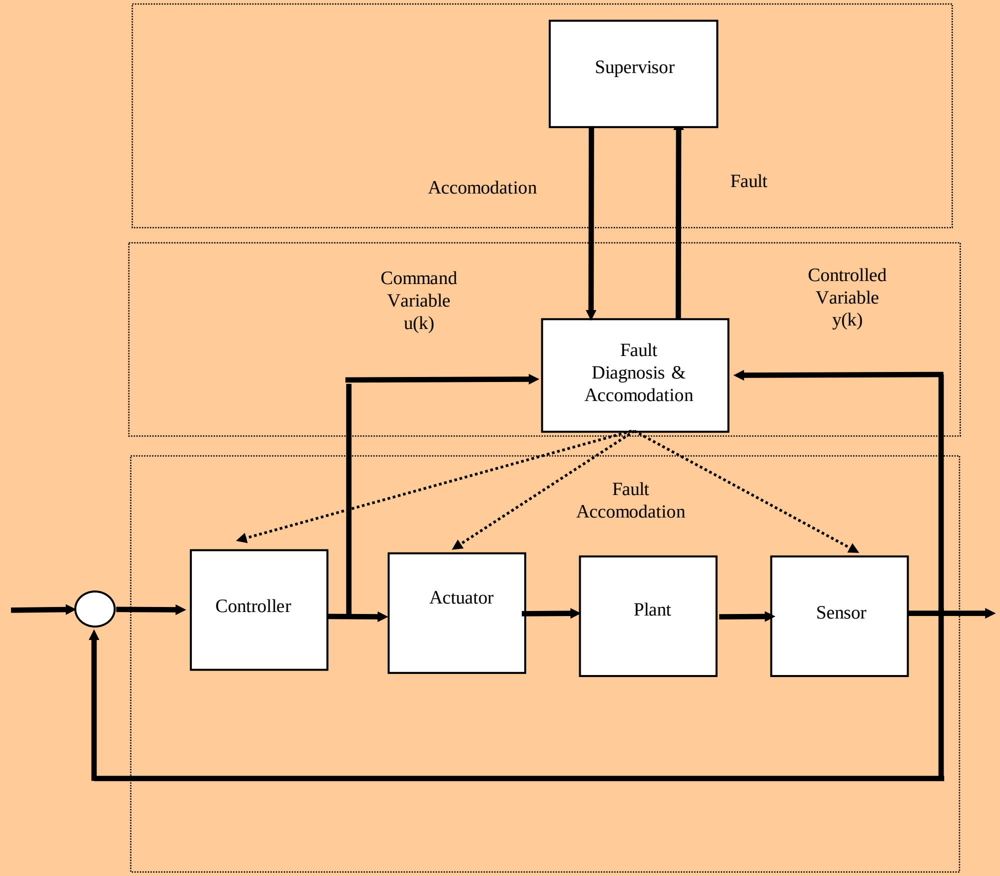
Fault Tolerant Control

Level 3
Supervision

Level 2
Fault Diagnosis
& Accomodation

Level 1
Control



Fault tolerant MPC problem

- The solution of a control problem consists on finding a control law in a given set of ***control laws*** $\mathcal{U}$ such that the controlled system achieves the ***control objectives*** $\mathcal{O}$ while its behavior satisfies a set of ***constraints*** $\mathcal{C}$.
- The solution of the problem is completely defined by the triple: $\langle \mathcal{U}, \mathcal{O}, \mathcal{C} \rangle$
- In the case of a linear constrained predictive control law:

$$\mathcal{O} : \min_{\tilde{u}} J(\tilde{x}, \tilde{u})$$

subject to:

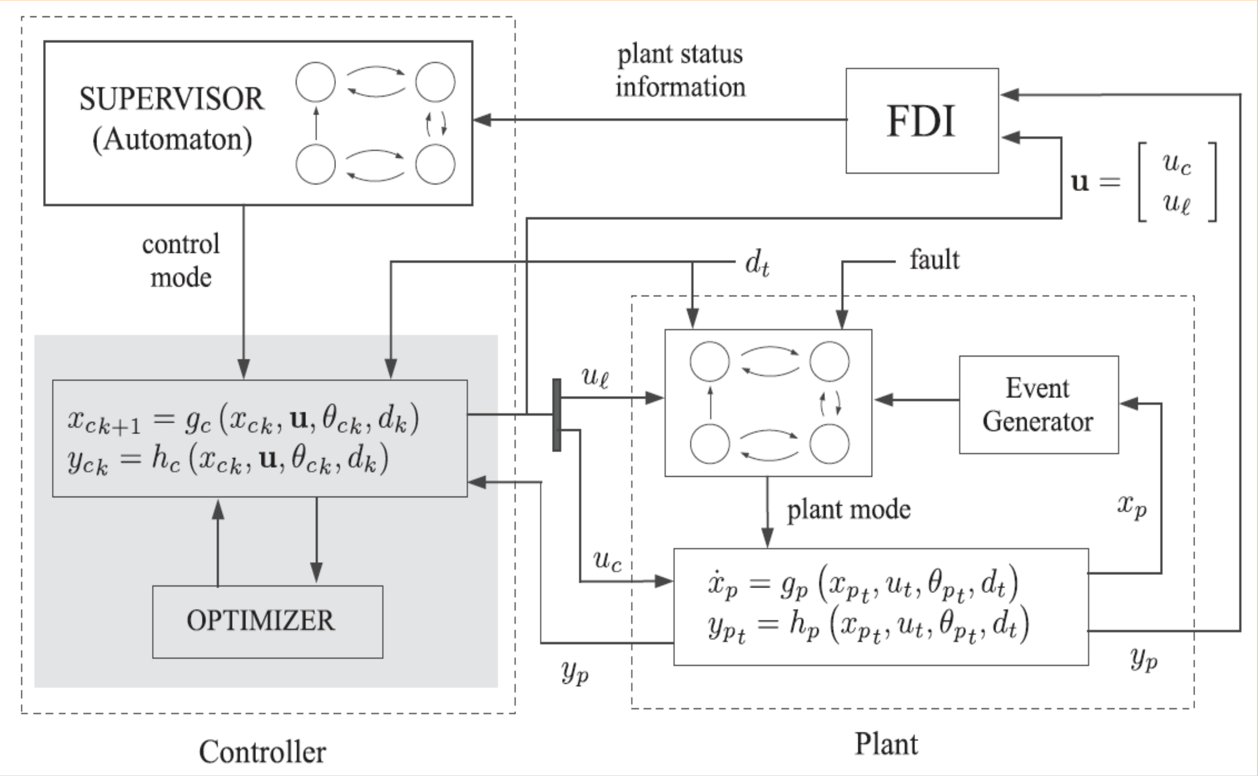
$$\mathcal{C} : \begin{cases} x_{k+1} = f(x_k, u_k) \\ u_k \in \mathcal{U} \quad k = 1, \cdots, N-1 \\ x_k \in \mathcal{X} \quad k = 0, \cdots, N \end{cases}$$

where:

$$\mathcal{U} = \left\{ u_k \in \mathbb{R}^m \middle| u_{min} \le u_k \le u_{min} \right\}$$
$$\mathcal{X} = \left\{ x_k \in \mathbb{R}^n \middle| x_{min} \le x_k \le x_{min} \right\}$$

$$\tilde{u}_k = (u_j)_0^{k-1} = (u_0, u_1, \cdots, u_{k-1})$$
$$\tilde{x}_k = (x_j)_0^{k-1} = (x_0, x_1, \cdots, x_k)$$

Hybrid MPC Fault-tolerant Control



Preliminary Definitions

- *Definition 1.* The ***feasible solution set*** is given by

$$\Omega = \left\{ \tilde{x},\tilde{u} \middle| \left(x_{k+1} = f(x_k, u_k) \right)_0^{N-1} \right\}$$

and gives the input and state sets compatible with system constraints which originate the set of predictive states.

- *Definition 2.* The ***feasible control objectives set*** is given by

$$J_\Omega = \left\{ J(\tilde{x},\tilde{u}) \middle| (\tilde{x},\tilde{u}) \in \Omega \right\}$$

and corresponds to the set of all values of *J* obtained from feasible solutions.

- *Definition 3.* The ***admissible solution set*** is given by $\mathcal{A} = \left\{ (\tilde{x},\tilde{u}) \in \Omega_f \middle| J(\tilde{x},\tilde{u}) \in \mathcal{J}_\mathcal{A} \right\}$

where Ω_f corresponds to the feasible solution set of a actuator fault configuration and $\mathcal{J}_\mathcal{A}$ defined as the admissible control objective set.

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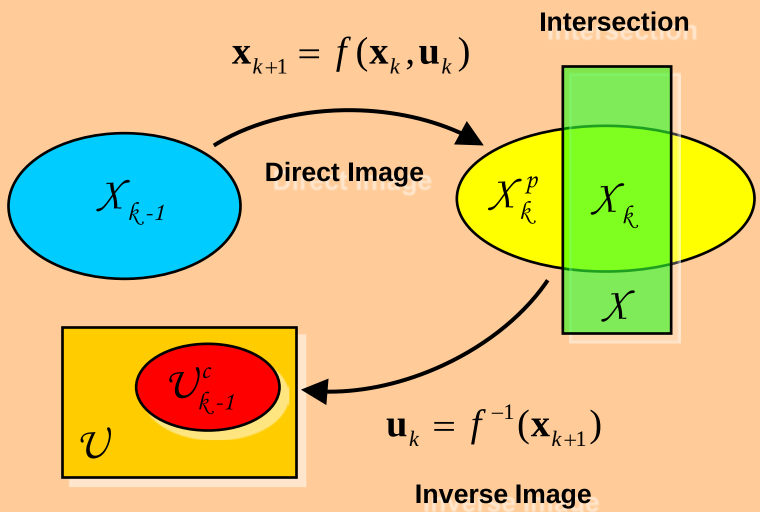
Admissibility Evaluation using Set Computations (1)

- The admissibility evaluation using a set computation approach starts obtaining the ***feasible solution set*** Ω given a set of initial states $\mathcal{X}_0$, the system dynamic and the system operating constraints over *N*.

Algorithm 1 Computation of Ω

$\mathcal{X}_k \Leftarrow \mathcal{X}_0$
$\Omega_0 \Leftarrow \mathcal{X}_0$
for $k = 1$ to N **do**
  $\mathcal{U}_{k-1} \Leftarrow \mathcal{U}$
  Compute $\mathcal{X}_k^p$ from $\mathcal{X}_{k-1}$ and $\mathcal{U}_{k-1}$
  Compute $\mathcal{X}_k^c = \mathcal{X} \cap \mathcal{X}_k^p$
  Compute $\mathcal{U}_{k-1}^c$ from $\mathcal{X}_k^c$
  $\Omega_k = \mathcal{X}_k^c \times \mathcal{U}_{k-1}^c$
  $\mathcal{X}_k \Leftarrow \mathcal{X}_k^c$
end for
$\Omega = \bigcup_{k=0}^{N} \Omega_k$



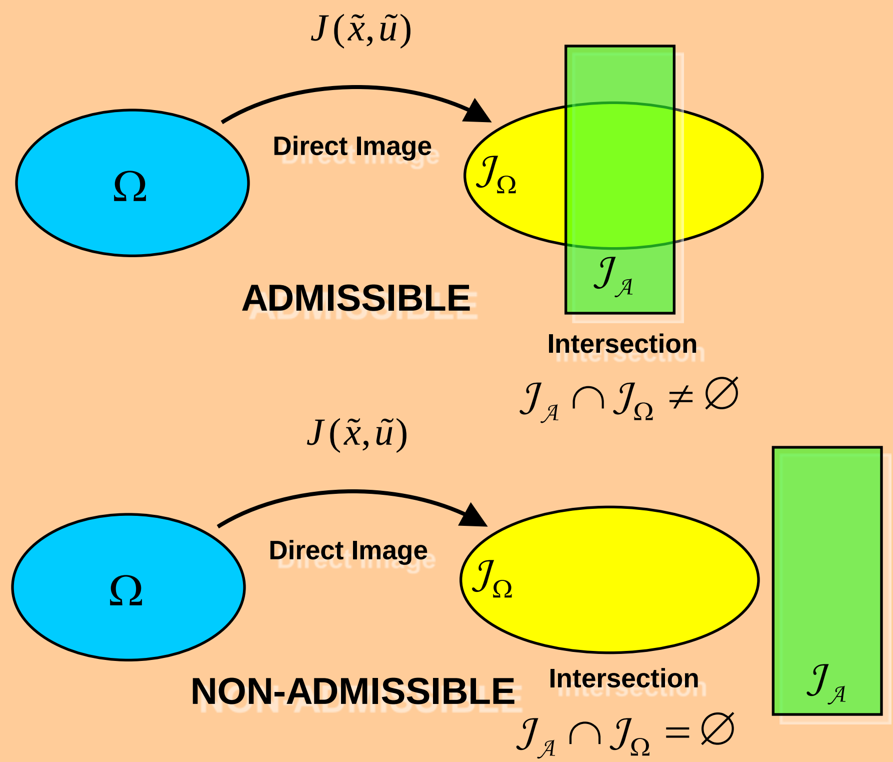
Puig, V. (2010). Fault diagnosis and fault tolerant control using set-membership approaches: Application to real case studies, *International Journal of Applied Mathematics and Computer Science*, *20*(4), 619-635. doi: https://doi.org/10.2478/v10006-010-0046-y

Admissibility Evaluation using Set Computations (2)

- At the same time that the ***feasible solution set*** is computed Ω , the ***feasible control objectives set*** $\mathcal{J}_\Omega$ at time *k*=*N* can be obtained using the following algorithm:

Algorithm 2 Computation of $\mathcal{J}_\Omega$ using Ω_k

$\mathcal{X}_k \Leftarrow \mathcal{X}_0$
$\Omega_0 \Leftarrow \mathcal{X}_0$
for $k = 1$ to N **do**
  Compute Ω_k *(See Algorithm 1)*
  Compute $\mathcal{J}_{\Omega_k}$ using $\Omega_k = \mathcal{X}_k^c \times \mathcal{U}_{k-1}^c$
end for
$\mathcal{J}_\Omega = \bigcup_{k=0}^{N} \mathcal{J}_{\Omega_k}$



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Admissibility Evaluation using Constraints Satisfactions (1)

- ***Constraints satisfaction problem***:

  "*A **constraints satisfaction problem** (CSP) on sets can be formulated as a 3-tuple H = (V,D,C) where:*

  - $V = \{ v_1 , \cdots ,v_n \}$ *is a finite set of variables,*
  - $D = \{D_1 , \cdots ,D_n \}$ *is the set of their domains represented by closed sets*
  - $C =\{c_1 , \cdots ,c_n \}$ *is a finite set of constraints relating variables of V "*

- A point solution of H is a n-tuple $(v_1 ,\cdots ,v_n)$ 2 D such that all constraints C are satisfied.
- The set of all point solutions of H is denoted by S(H). This set is called the global solution set.
- The variable v_i 2 V_i is consistent in H if and only if:

$$\forall v_i \in \mathcal{V}_i \ \exists \ (\tilde{v}_1 \in \mathcal{D}_1, \cdots , \tilde{v}_n \in \mathcal{D}_n) \ |(\tilde{v}_1, \cdots , \tilde{v}_n) \in \mathcal{S}(\mathcal{H})$$

with *i=1...n*

Admissibility Evaluation using Constraints Satisfaction (2)

- The admissibility evaluation requires the computation of the admissible solution set:

$$\Omega = \left\{ \tilde{x}, \tilde{u} \middle| \left(x_{k+1} = f(x_k, u_k) \right)_0^{N-1} \right\}$$

- Its definition suggests a way of implementation since its mathematical description can be viewed as a constraints satisfaction problem:

Algorithm 1: Admissibility Evaluation using Constraints Satisfaction

At each time instant k over N, the feasible solution set is determined by solving the CSP $\mathcal{H} = (\mathcal{V}, \mathcal{D}, \mathcal{C})$ associated with the constraints $\mathcal{C}$ of the CNMPC problem,where

$$\begin{aligned}
\mathcal{V} &= \{\overbrace{x_1, x_2, \cdots, x_N}^{\tilde{x}}, \overbrace{u_1, u_2, \cdots, u_{N-1}}^{\tilde{u}}, J\} \\
\mathcal{D} &= \{\mathcal{X}_1, \mathcal{X}_2, \cdots, \mathcal{X}_N, \mathcal{U}_1, \mathcal{U}_2, \cdots, \mathcal{U}_{N-1}, \mathcal{J}_{\mathcal{A}}\} \\
\mathcal{C} &= \Bigg\{ \Big(x_{k+1} = f(x_k, u_k) \Big)_0^{N-1}, \\
&\qquad J(\tilde{x}, \tilde{u}) = \phi(x_N) + \sum_{i=0}^{N-1} \Phi(x_i, u_i) \Bigg\}
\end{aligned}$$

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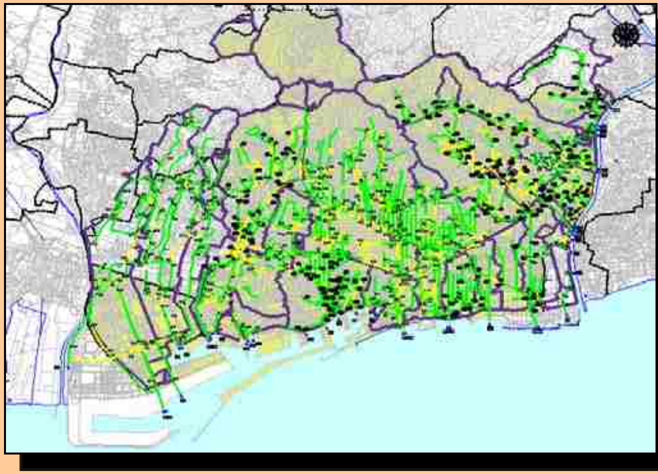
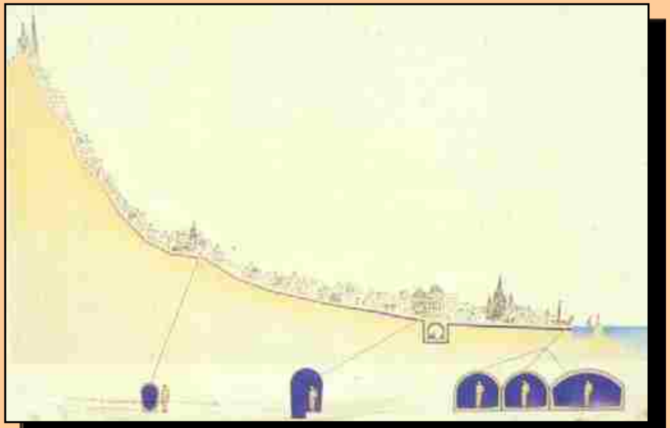
The Barcelona Sewer Network

- Data
  - Typology: combined
  - Length: 1.650 km
  - Storage capacity: 2.634.124 m^3
  - Visitable portion: 55,12%
  - Mean transversal section: 1,8 m^2
  - 31 catchment area: 12.326 ha

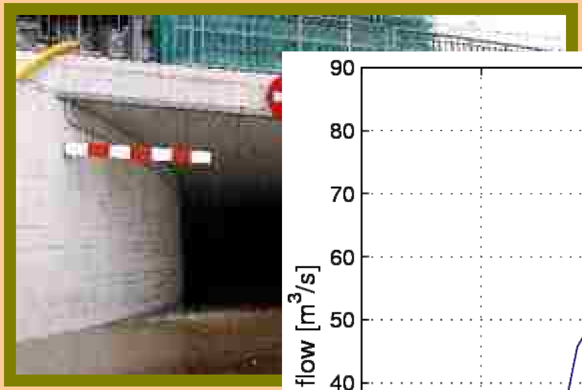


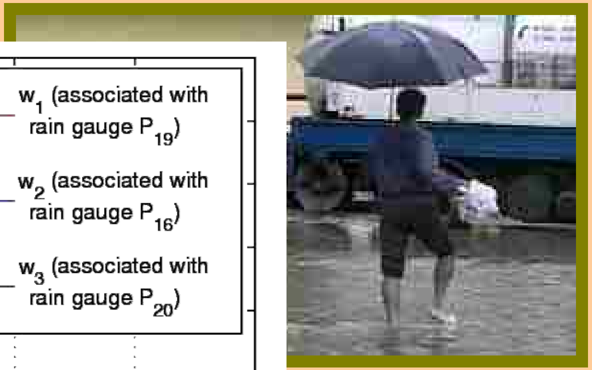
- Particularities
  - Topographic profile: steep slope, gentle at rivers and sea
  - Urban ground: 90% impervious
  - Meteorology: yearly precipitation: 600mm, intensity: up to 150 mm/h in 15 minutes

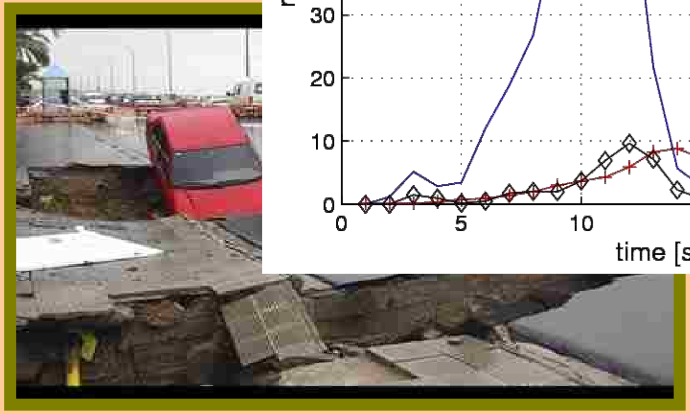


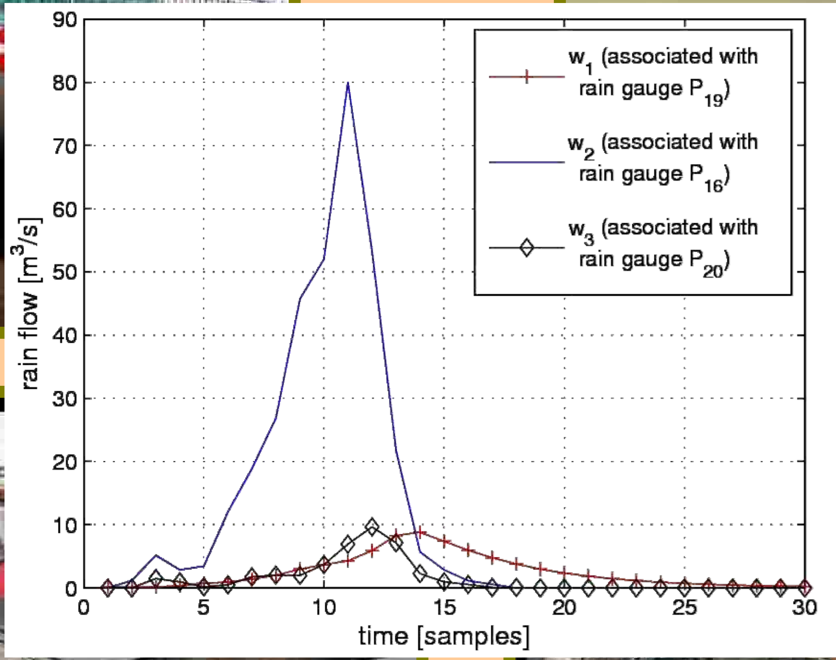
C. Ocampo-Martinez, V. Puig, G. Cembrano and J. Quevedo, "Application of predictive control strategies to the management of complex networks in the urban water cycle [Applications of Control]," in IEEE Control Systems Magazine, vol. 33, no. 1, pp. 15-41, Feb. 2013, doi: 10.1109/MCS.2012.2225919.

Barcelona and its Rain









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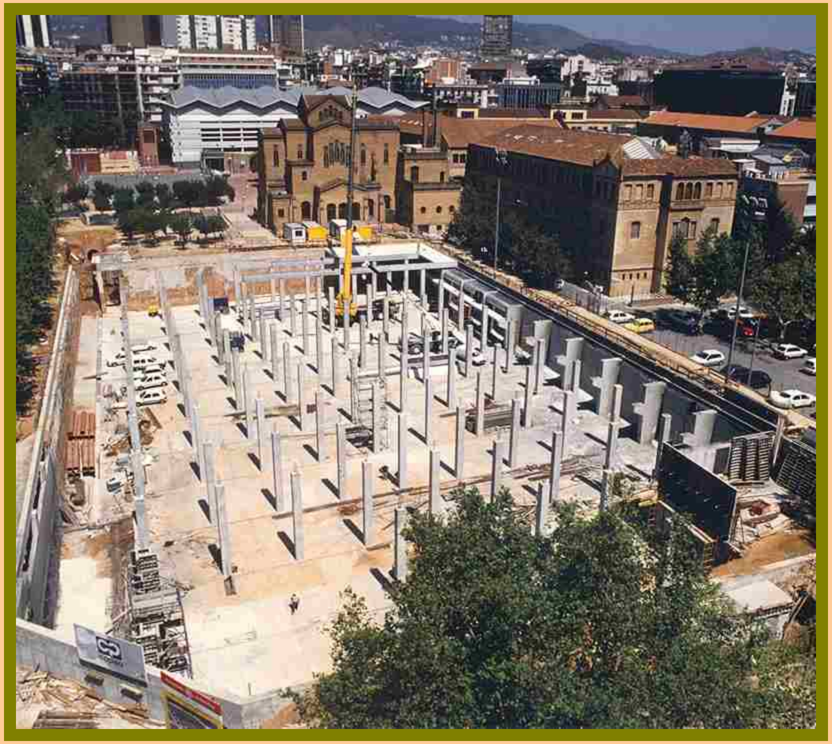
Solution (1): Detention Tanks



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Solution (2): Barcelona’s RTC System

ELEMENTS	NUMBER
Rain gauges	22
Water level sensors	119
Pumping Stations	11
Gates	23
Detention Tanks	10

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MPC Multicriteria optimization

$$J = \sum_{k=0}^{N-1} (\alpha J_{flood}{}^{k} + \beta J_{CSO}{}^{k} + \gamma J_{WWTP}{}^{k})$$

- *Reduction of the risk of floods*

$$J_{flood} = \sum_{j} \max(o, q_j - \lim_{q_j})$$

q_j flow through sewer *j*

- *Environment protection*

$$J_{cso} = \sum_{l} CSO_l$$

CSO_l^k combined sewer overflow volume at site *l*

- *Optimization of the WWTP*

$$J_{WWTP} = \sum_{i} (WWTP_i - WWTP^*{}_i)$$

$WWTP_i$ waste water treatment plant flow *i*

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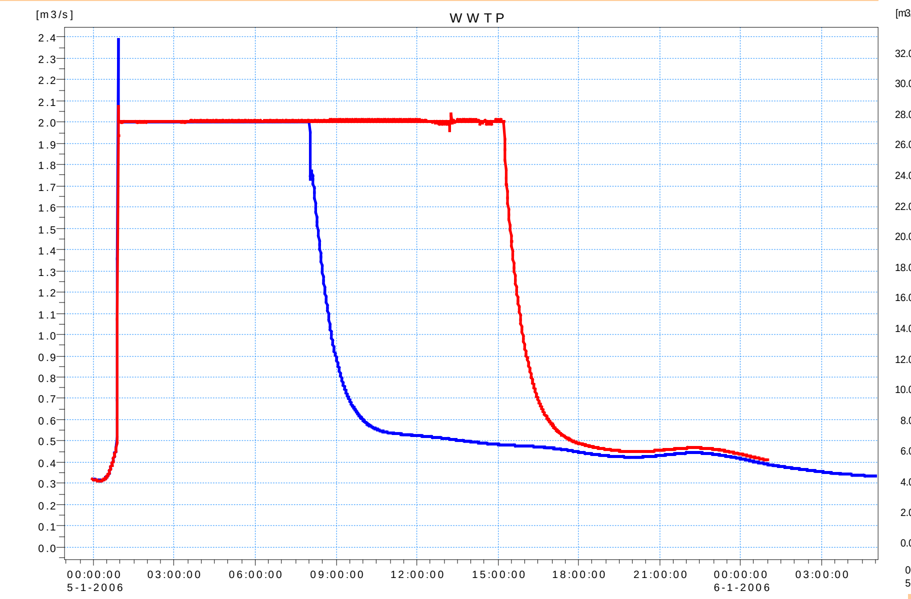
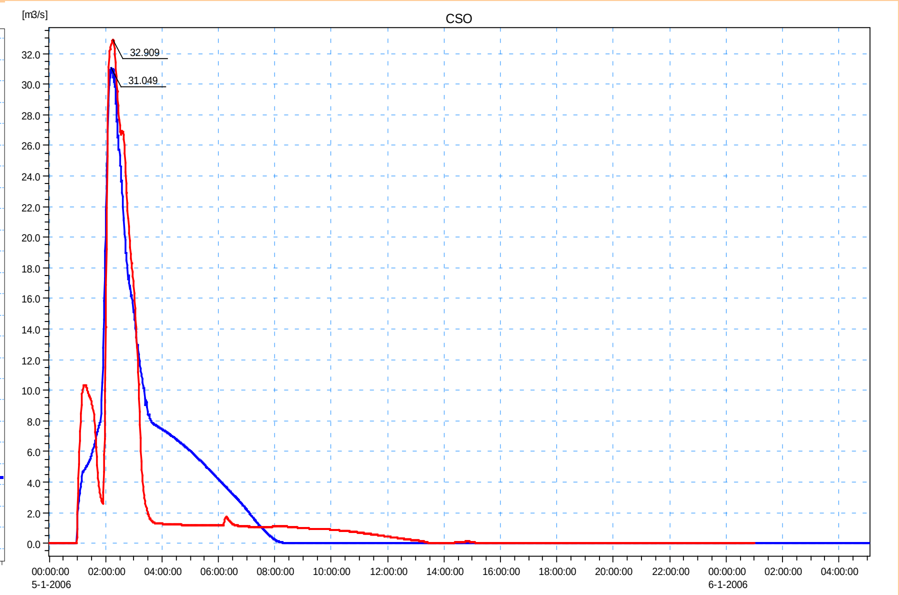
Global Control vs Local control

WWTP Volume



CSO Volume

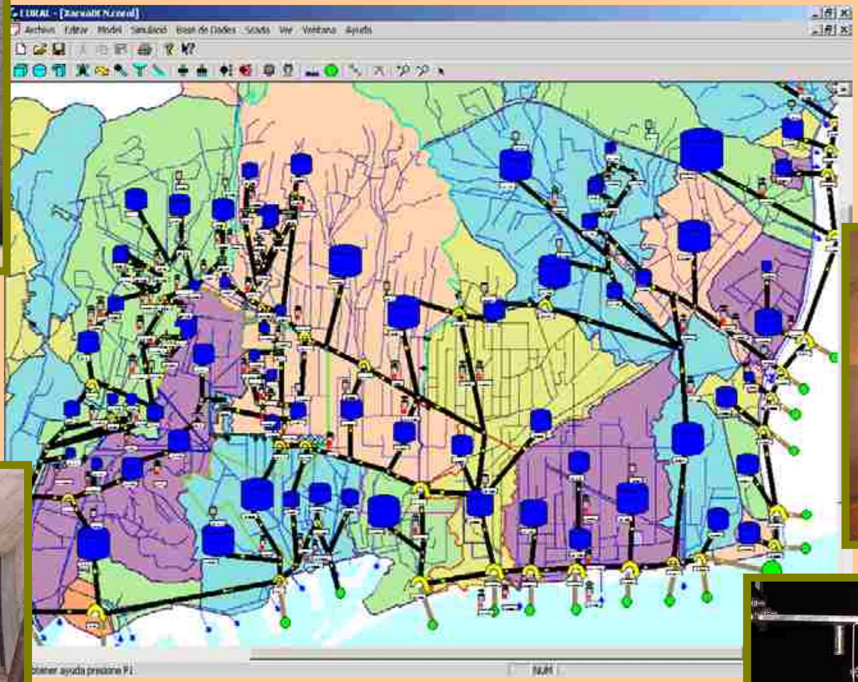


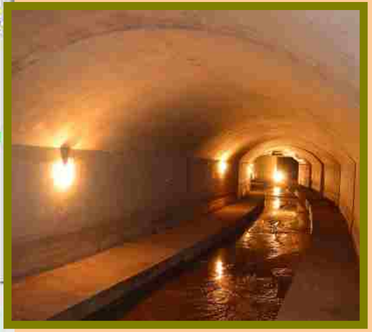
50 % improvement

-18 % reduction

Blue: Local Control
Red: Global Control

CORAL: MPC tool for Sewer Networks





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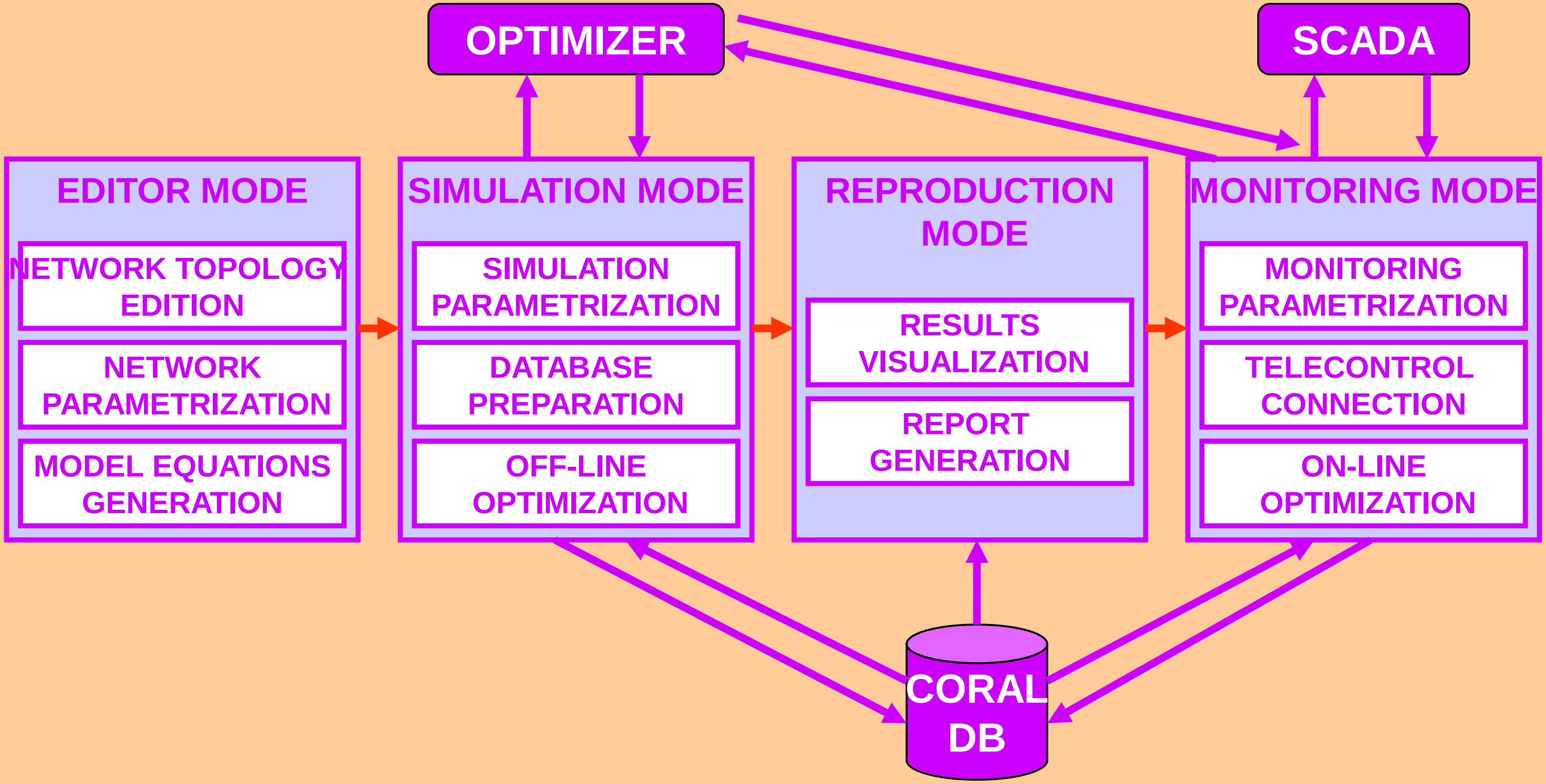
CORAL Architecture



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Introduction to FDI in Sewer Networks

- In this presentation, the FDI problem of rain gauges and limnimeters of Barcelona’s urban sewer system is addressed.
- Rain gauges and limnimeters are used for the real-time global control of the whole Barcelona network.
- Often these instruments are out of order in rain scenarios when the control system must be fully operative.
- In order to detect and isolate faulty instruments and to reconstruct faulty measurements from data fusion, a fault diagnosis system is necessary.

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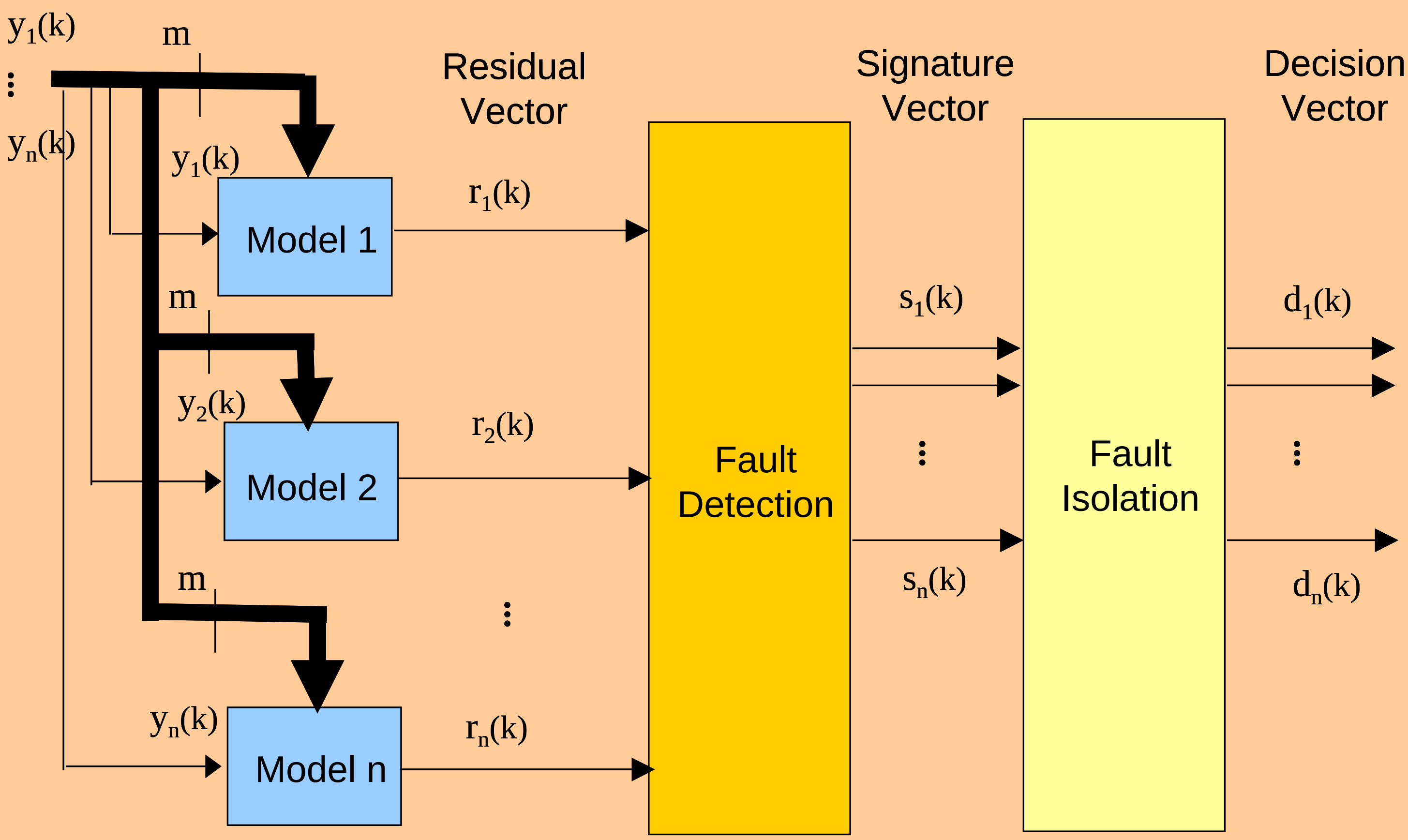
V. Puig, J. Blesa, Limnimeter and rain gauge FDI in sewer networks using an interval parity equations based detection approach and an enhanced isolation scheme, Control Engineering Practice, Volume 21, Issue 2, 2013, Pages 146-170, https://doi.org/10.1016/j.conengprac.2012.09.015.

The Architecture of the FDI System



Fault Isolation Procedure

Signature Vector $s(k)$: $\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$

$$d_i(k) = distHam(s(k), S_i)$$

Decision Vector $d(k)$: $\begin{bmatrix} 2 \\ 0 \\ 3 \\ 1 \\ 2 \\ 2 \\ 3 \end{bmatrix}$

$$f(k) = arg\,min(d(k))$$

Fault Indicator: $f(k)=y2$

Hamming Distance: number of different bits between two binary codes

$$distHam = c_1 \oplus c_2$$

Signature Matrix

	y1	y2	y3	y4	y5	y7	y8
r1	1	0	1	0	0	0	1
r2	1	1	0	1	0	0	0
r3	1	0	1	0	0	1	0
r4	0	0	0	1	1	0	1

$S_i(k)$

$$r_1(k) = y_1(k) - a_{13}y_3(k) - a_{18}y_8(k)$$
$$r_2(k) = y_2(k) - a_{21}y_1(k) - a_{24}y_4(k)$$
$$r_3(k) = y_3(k) - a_{31}y_1(k) - a_{37}y_7(k)$$
$$r_4(k) = y_4(k) - a_{45}y_5(k) - a_{48}y_8(k)$$

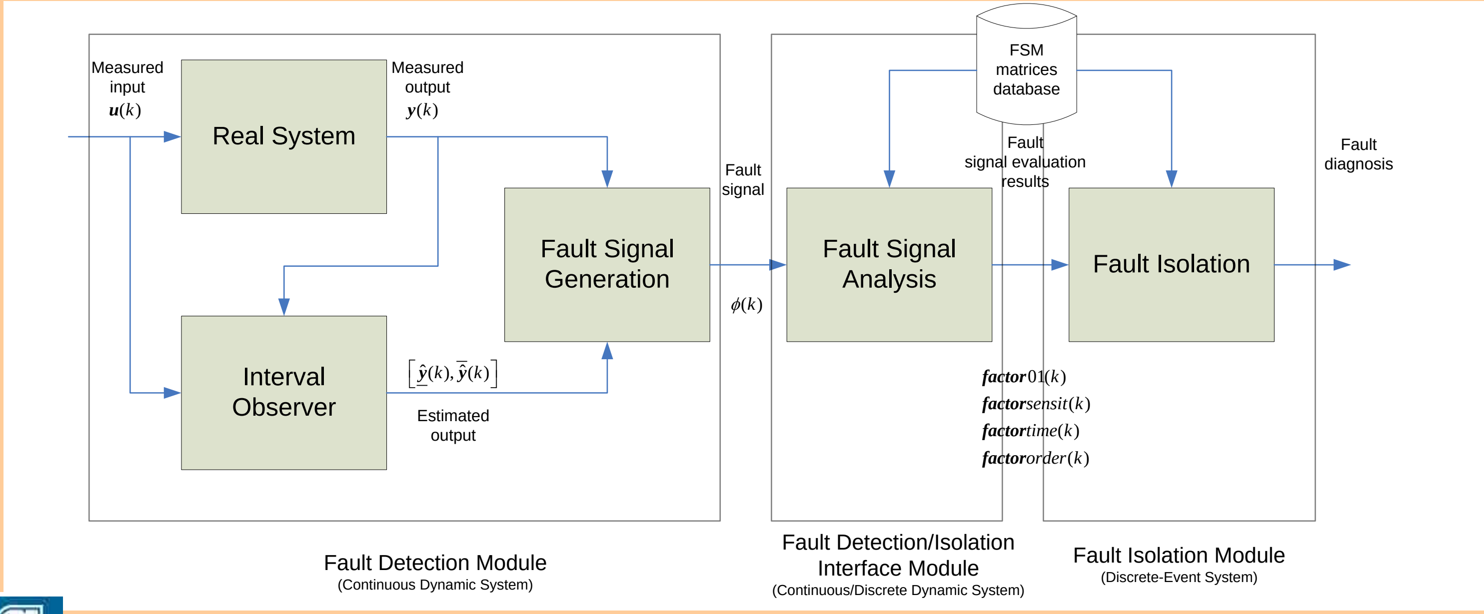
Usual assumptions

- Single fault and exoneration
- If multiple faults, no compensation

Enhanced Fault Isolation Scheme

In particular, such interface can be improved taking into account the following information:

- ***residual value size***: big violation of the threshold or only a small fault signal activation.
- ***residual sensitivity*** with respect to a certain fault.
- ***time pattern*** of fault signal occurrence.
- ***order*** of fault signal occurrence.



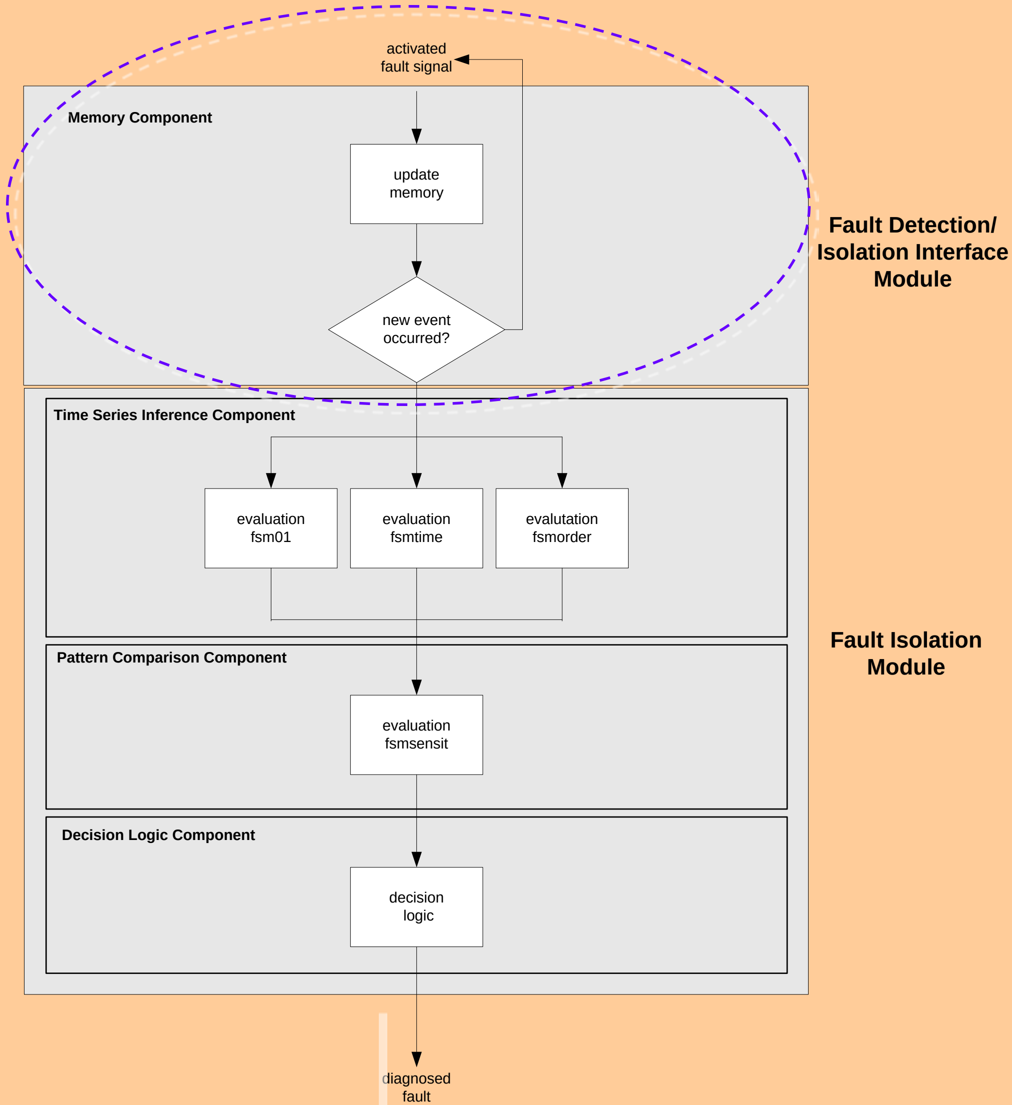
Interface between Fault Detection and Isolation Modules

- The interface is based on a memory implemented as a table in which events in the residual history are stored:

$$\phi_i(k) = \begin{cases} \dfrac{(r_i^o(k)/\overline{r}_i^o(k))^4}{1+(r_i^o(k)/\overline{r}_i^o(k))^4} & if \quad r_i^o(k) \geq 0 \\[2ex] -\dfrac{(r_i^o(k)/\underline{r}_i^o(k))^4}{1+(r_i^o(k)/\underline{r}_i^o(k))^4} & if \quad r_i^o(k) < 0 \end{cases}$$

- For each row, the first column stores the occurrence time t_i, the second one stores, the $\phi_{i,max}$, and the third one stores the sign of the residual.

- If the fault detection component detects a new fault signal, it updates the memory by filling out the three fields.



Fault Detection and Isolation Interface: *FSM* Matrices

- It is based on the concept of the theoretical ***fault signature matrix*** (FSM) which was introduced by (Gertler, 1998).

- This matrix stores the theoretical binary influence of a given fault f_j (column of ***FSM***) on a given residual $r_i(k)$ or equivalently, on a given fault signal $\phi_i(k)$ (row of ***FSM***).

- Here, the fault signature matrix concept is generalized since the binary interface is extended taking into account more fault signal properties.

Fault Signal Properties	***FSM*** *Matrix*
Binary	***FSM*** *01*
Sign	***FSM*** *sign*
Fault residual sensitivity	***FSM*** *sensit*
Occurrence order	***FSM*** *order*
Occurrence time instant	***FSM*** *time*

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Limnimeter Modelling (1): "Virtual Reservoir Approach"

- Propagation of flows through sewer pipes can be described by numerical solution of the continuity and momentum Saint-Vennant's partial differential equations.

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = 0$$

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right) + gA\frac{\partial y}{\partial x} - gA\left(I_0 - I_f\right) = 0$$

- Saint-Vennant's equations can be used to perform simulation studies but are highly complex to solve in real-time, specially for large scale systems.

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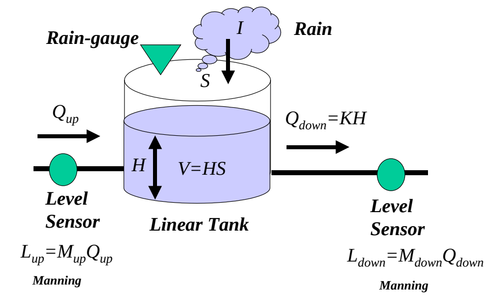
Limnimeter Modelling (2):
"Virtual Reservoir Approach"

- The sewerage network is modeled through a simplified graph relating the main sewers and set of virtual and real reservoirs.

- A virtual reservoir is an aggregation of a catchment of the sewage network which approximates the hydraulics of rain, runoff and sewage water retention thereof.

- The hydraulics of virtual reservoirs are:

$$\frac{dV(t)}{dt} = Q_{in}(t) + I(t)S - Q_{out}(t)$$

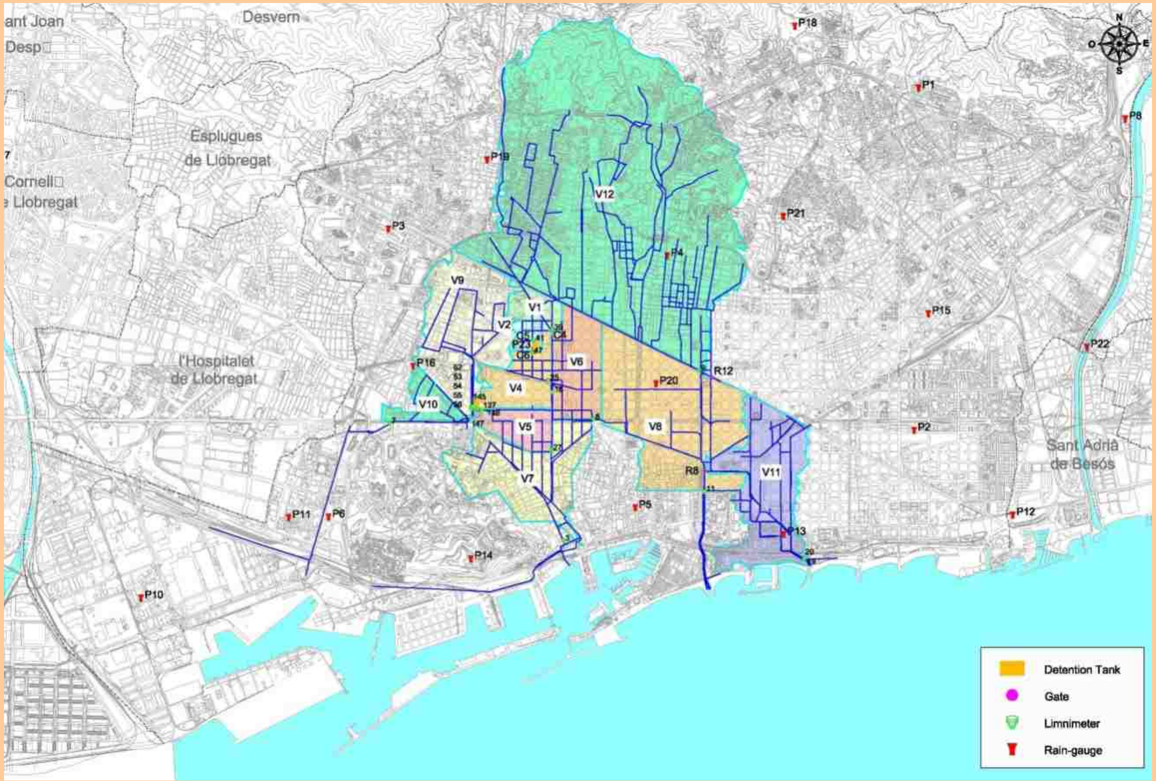


Using Manning's formula and discretising:

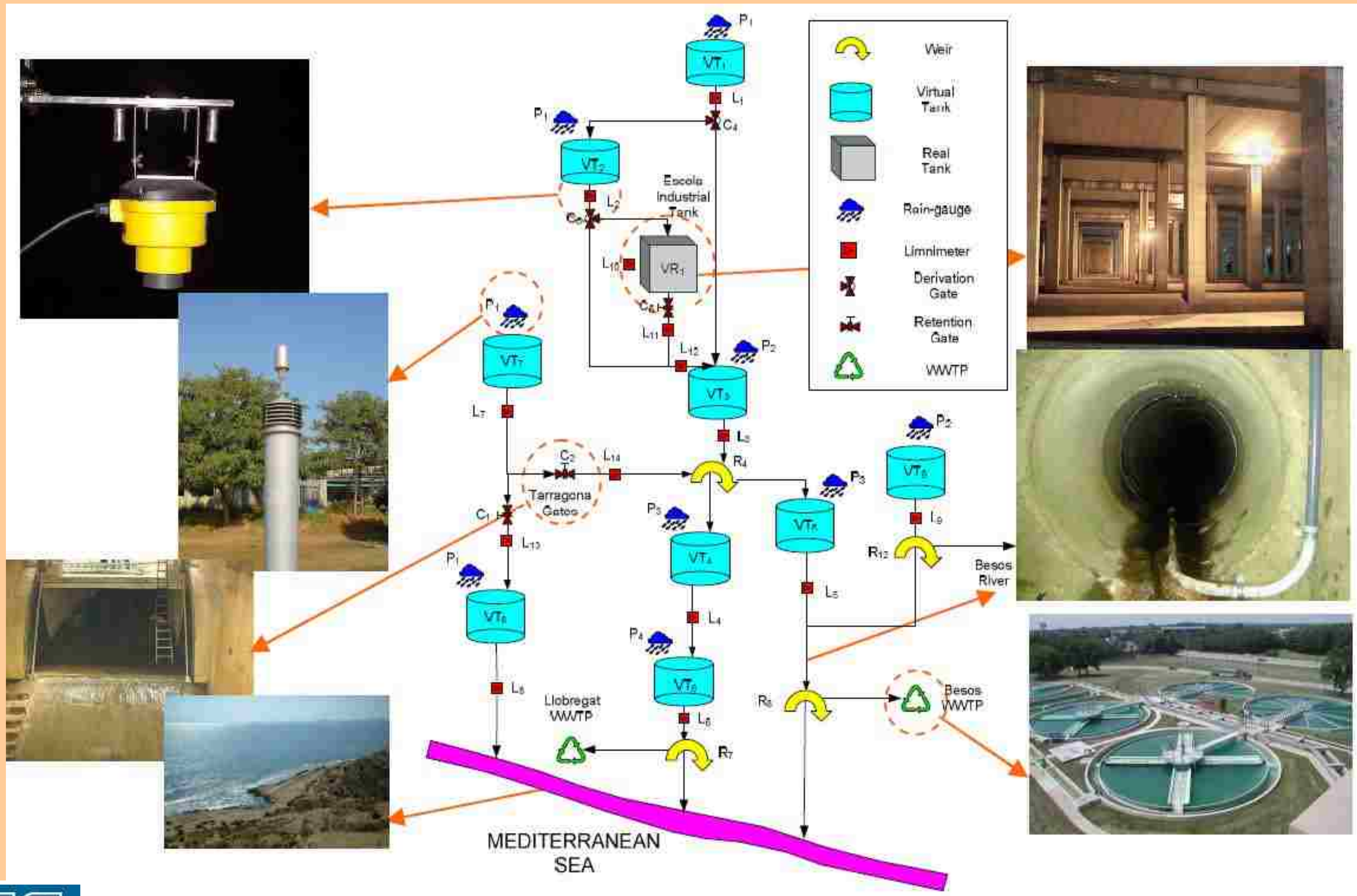
$$Q_{up}(t) = M_{up} L_{up}(t)$$
$$Q_{down}(t) = M_{down} L_{down}(t)$$

$$L_{down}(k+1) = aL_{down}(k) + bL_{up}(k) + cI(k))$$

Application Example (1): Modeling Barcelona Sewer Network using Virtual Tanks



Application Example (2): Modeling Barcelona Sewer Network using Virtual Tanks



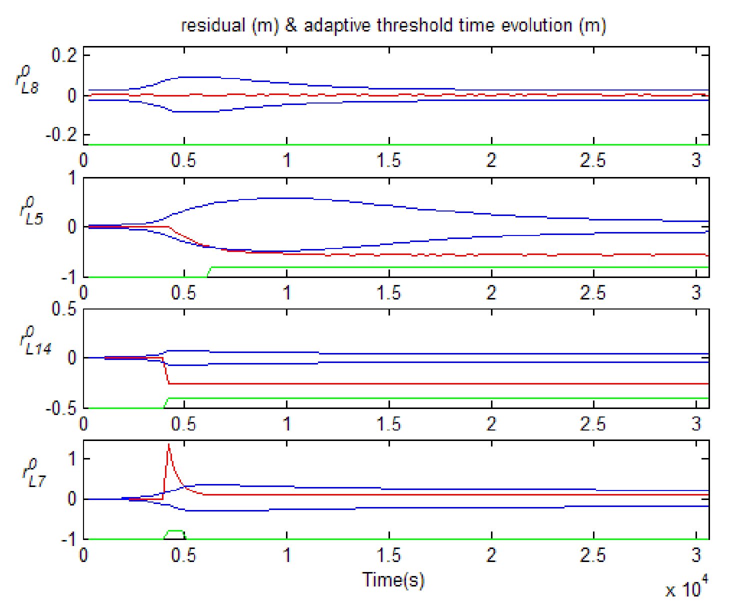
Application Example: Structure of the Limnimeter Models

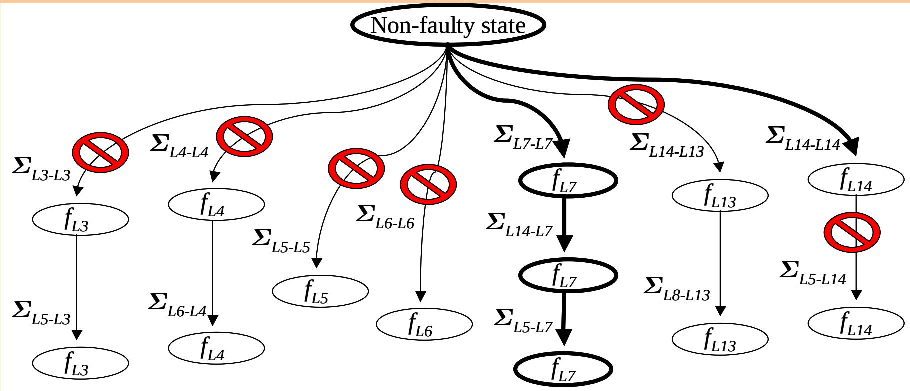
- Applying the limnimeter modelling methodology based on “virtual tanks” to the considered sewer network:
  - 12 limnimeters are modelled allowing to compute 12 residuals.
  - Faults affecting 14 limnimeters can be diagnosed.

	L_1	L_2	L_3	L_4	L_5	L_6	L_7	L_8	L_9	L_{10}	L_{11}	L_{12}	L_{13}	L_{14}	P_1	P_2	P_3	P_4
L_1	X														X			
L_2	X	X													X			
L_3			X									X				X		
L_4				X													X	
L_5			X		X		X							X			X	
L_6				X		X												X
L_7							X								X			
L_8								X					X		X			
L_9									X							X		
L_{10}		X								X	X							
L_{12}											X	X						
L_{14}							X						X	X				

Application Example: Fault Scenario affecting L_7

- A fault affecting limnimeter L_7 occurs at t_0= *4000s*.





Residual time evolution

Fault Tolerant Control

Application Example (1)

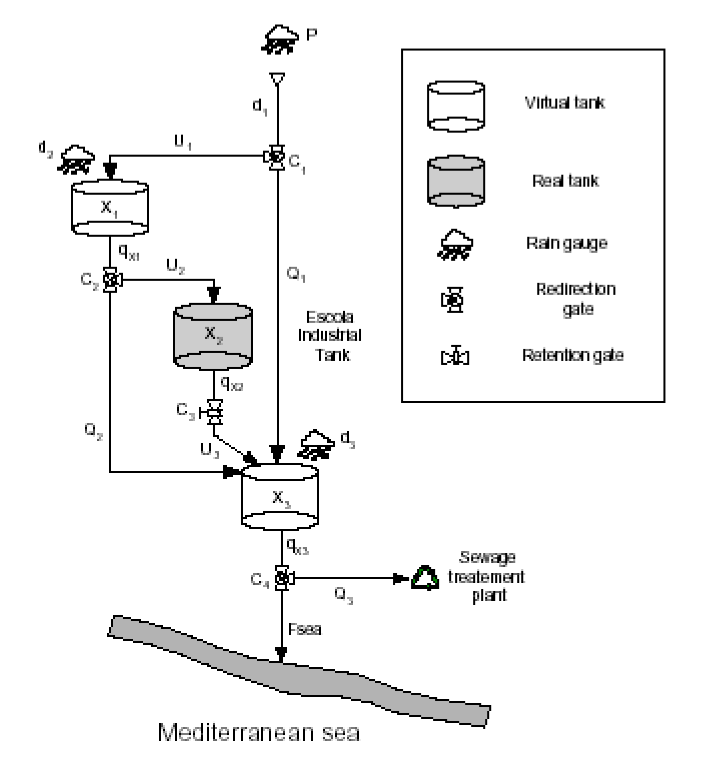
- Consider the system corresponding to a piece of Barcelona sewer network described by the discrete-time state equations

$$x_{k+1} = Ax_k + Bu_k + B_p d_k$$

where:

$$A = \begin{bmatrix} 1-\Delta t\beta_1 & 0 & 0 \\ 0 & 1 & 0 \\ \Delta t\beta_1 & 0 & 1-\Delta t\beta_3 \end{bmatrix}$$

$$B = \Delta t \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ -1 & -1 & 1 \end{bmatrix} \qquad B_p = \Delta t \begin{bmatrix} 0 & \alpha_2 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & \alpha_3 \end{bmatrix}$$



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Application Example (2)

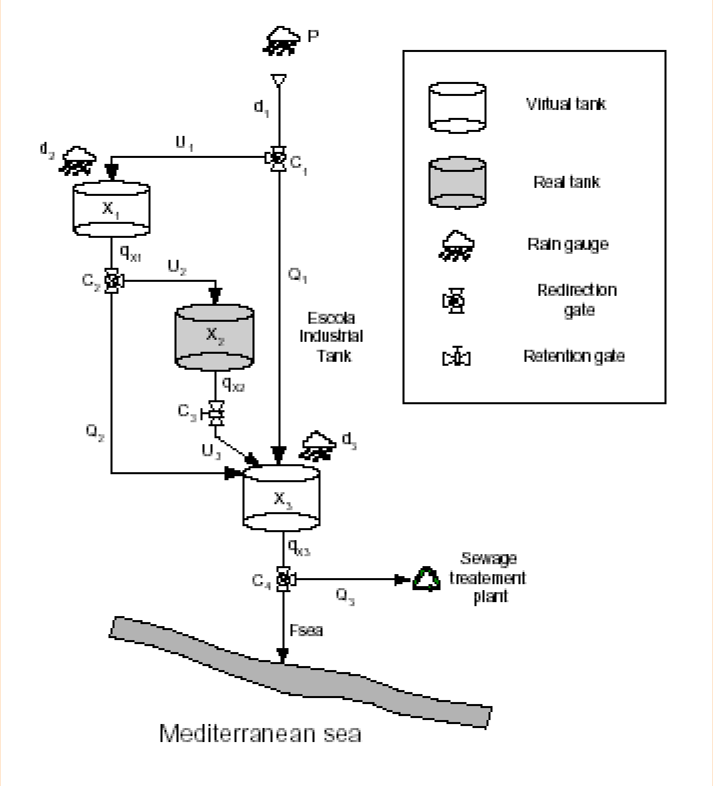
- The systems constraints are:

  - ***Bounding constraints***: refers to physical restrictions.

$$x_{1,k} \in [0,\infty] \qquad u_{1,k} \in [0,11]$$
$$x_{2,k} \in [0,35000] \qquad u_{2,k} \in [0,25]$$
$$x_{3,k} \in [0,\infty] \qquad u_{3,k} \in [0,7]$$

  - ***Mass conservation constraints:***

$$d_{1,k} = u_{1,k} + Q_1(k)$$
$$q_{x_1,k} = u_{2,k} + Q_2(k)$$
$$q_{x_2,k} \geq u_{3,k}$$



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Reconfiguration Case

- This case considers actuators completely closed or completely open due to the fault, what would change the admissibility of the obtained actuator fault configurations.

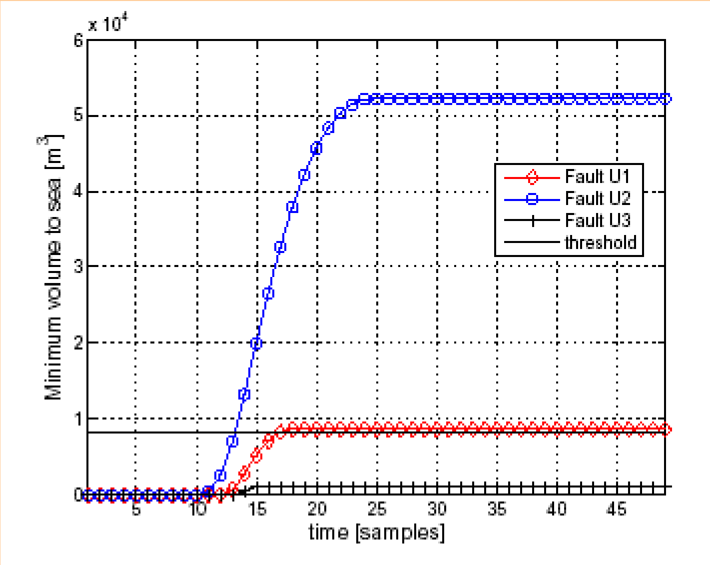

TABLE I

ADMISSIBILITY OF FAULT CONFIGURATIONS FOR POLLUTION - RECONFIGURATION

Fault Location	Min. Volume $[m^3]$	Admissibility Status
No fault	1050	—
Fault in u_1	8800	No Admissible
Fault in u_2	52200	No Admissible
Fault in u_3	1050	Admissible

ADMISSIBILITY CRITERIA: $V_{sea}^{f} \geq 8V_{sea}^{o}$

Accomodation Case

- This case considers that faults produces the reduction of the actuators operating range (for example from 0-100\% to 0-50\%).

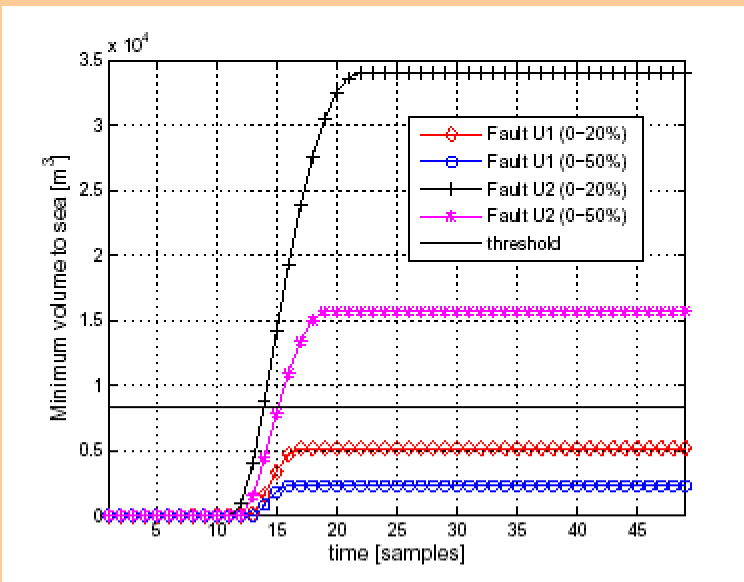

TABLE II

ADMISSIBILITY OF FAULT CONFIGURATIONS - ACCOMMODATION

Fault Location	Operation range	Min. Volume $[m^3]$	Admissibility Status
No fault	—	1050	—
Fault in U_1	0-20%	5200	Admissible
Fault in U_1	0-50%	2300	Admissible
Fault in U_2	0-20%	34000	No Admissible
Fault in U_2	0-50%	15700	No Admissible

ADMISSIBILITY CRITERIA: $V_{sea}^{f} \geq 8V_{sea}^{o}$

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Conclusions (1)

- This presentation has reviewed the use of set-membership methods in robust fault detection and isolation (FDI) and tolerant control (FTC).
- Alternatively to the statistical methods, set-membership methods use a deterministic unknown-but-bounded description of noise and parametric uncertainty (interval models).
- Using approximating sets to approximate the set of possible behaviours (in parameter or state space), these methods allows to check the consistency between observed and predicted behaviour.
- When an inconsistency is detected a fault can be indicated, otherwise nothing can be stated.

Conclusions (2)

- The same principle has been used to estimate interval models for fault detection and to develop methods for fault tolerance evaluation.

- Finally, same real application of these methods has been used to exemplify the successful uses in FDI/FTC.

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Further Research

- As further research, the set-membership approach could be extended to:

- extension to non-linear systems via the use of LPV models.
- deal with the fault isolation and estimation tasks exploiting the set arithmetic concepts
- adaptive thresholding in the the frequency domain
- better understand the links between the set-membership and interval approach revised in this presentation
- further extend their application to fault tolerant control as means to specify admissible closed loop behaviours.

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Thank you very much
for your attention!!!

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